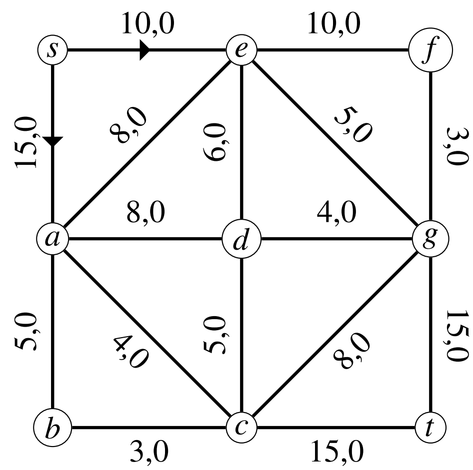
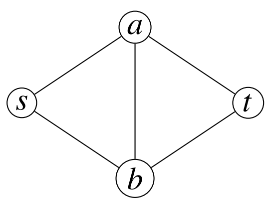
1. In the network below, the capacity and flow value for each edge are represented with an ordered pair. Since the flow is everywhere zero, there is no need to direct the edges. However two edges are directed so you can see how to change the diagram. Find a maximum flow from s to t. Prove your answer is optimal by finding a cut with minimum capacity.



2. Given $n \in \mathbb{N}$, find a capacity function for the network below such that the algorithm from the proof of the max-flow min-cut theorem (aka Ford-Fulkerson Theorem) will need more than n augmenting paths if the algorithm consistently chooses the path badly.
An **augmenting** path is an st-path such that every (directed) edge on the path has available capacity. That is $c(\overrightarrow{e}) > f(\overrightarrow{e})$.



3. Use the max-flow min-cut theorem to prove part 1 of Corollary 3.3.5. (Copied below.)

> Let a and b be two distinct vertices of the simple graph G. If $ab \notin E$, then the minimum number of vertices separating a from b is equal to the maximum number of independent ab-paths in G.

4. Determine the value of $ex(n, K_{1,r})$ for all r and n. (Assume that $n > r$.)

5. Show that every connected graph on at least three vertices contains a path or a cycle of length at least $\min\{2\delta, |G|\}$.

6. Prove the Erdös-Sös Conjecture for the case when the tree is a path. (Hint: use the previous problem.