MATH 663
Fall 2017
Midterm

Name:

Books and notes are not allowed. There are 9 problems worth a total of 100 points. You have two hours to complete the exam.

1. (10 points) Determine whether or not the sequence $5, 5, 4, 4, 3, 3, 2, 1, 1$ is graphic. If the sequence is graphic, demonstrate a graph with this degree sequence. If the sequence is not graphic, give a well-defended argument that it is not. Reference any theorems you are using.

2. (10 points) In the matrix below, $a_{i,j}$ gives the weight of the edge between x_i to y_j for a complete bipartite graph $K_{5,5}$. Use the Hungarian Algorithm to find a maximum weight matching (or a transversal of maximum sum) in this graph. Prove that your answer is correct by exhibiting a solution to the dual problem.

$$\begin{pmatrix} 3 & 5 & 5 & 3 & 6 \\ 5 & 6 & 4 & 2 & 7 \\ 1 & 3 & 4 & 4 & 5 \\ 7 & 6 & 5 & 5 & 7 \\ 5 & 3 & 5 & 4 & 1 \end{pmatrix}$$

3. (a) (2 points) Define *strong component* in a digraph.

   (b) (3 points) Give an example of a directed graph on 4 vertices with exactly 3 strong components.

   (c) (2 points) Define a *tournament*.

   (d) (3 points) Explain why it is not possible to construct a tournament on 4 vertices with exactly 3 strong components.

4. (10 points) Find a tree with Prüfer Code 2,6,1,9,4,4,9,2.

5. (a) (2 points) State the definition of a *tree*.

(b) (4 points) Give an example of a tree T on 6 vertices and a set $S \subseteq V(T)$, such that S is a maximal independent set of vertices that is *not* a maximum independent set of vertices.

(c) (4 points) Draw all nonisomorphic trees on 6 vertices with maximum degree 3. Do not draw any isomorphism class more than once.

6. (10 points) Prove that every simple graph on at least two vertices has at least two vertices of the same degree.

7. (16 points) For $K_{m,n}$ and P_n, find the following. Just plain answers without justification are acceptable here.

   (a) the radius of the graph.

   (b) the diameter of the graph

   (c) the center of the graph

   (d) the number of edges in a maximum matching.

8. (12 points) Assume every edge of a K_n has been assigned some nonnegative integer weight. Prove that the total weight on every cycle is even if and only if the total weight on every triangle is even. (Note: Make the logical structure of your argument clear.)

9. (a) (2 points) Define $\alpha(G)$, the independence number of the graph G.

(b) (2 points) Explain what the symbol $\Delta(G)$ means.

(c) (8 points) Prove that for every graph G, $\alpha(G) \geq \frac{n(G)}{\Delta(G)+1}$.