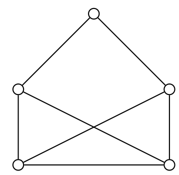
MATH 663
Spring 2010
In-Class Final Exam

Books and notes are not allowed. There are eight problems each worth 10 points. You have two hours to complete the exam.

1. (10 points) Let G be the graph drawn on the right: 

   (a) Determine $\chi'(G)$. (This requires explanation. Clearly identify any theorems you apply.)

   (b) Determine $L(G)$, the line graph of G.

2. (10 points) Prove that G has a Hamiltonian path only if for every $S \subseteq V(G)$, the number of components of $G - S$ is at most $|S| + 1$.

3. (10 points)

   (a) State Euler's Formula.

   (b) Prove that the Petersen graph is nonplanar by using Euler's Formula and the fact that the Petersen graph has girth 5.

4. (10 points) Prove that if G is a simple graph with $\delta(G) = k \geq 2$, then G contains a cycle with at least $k + 1$ vertices.

5. (10 points) Examples

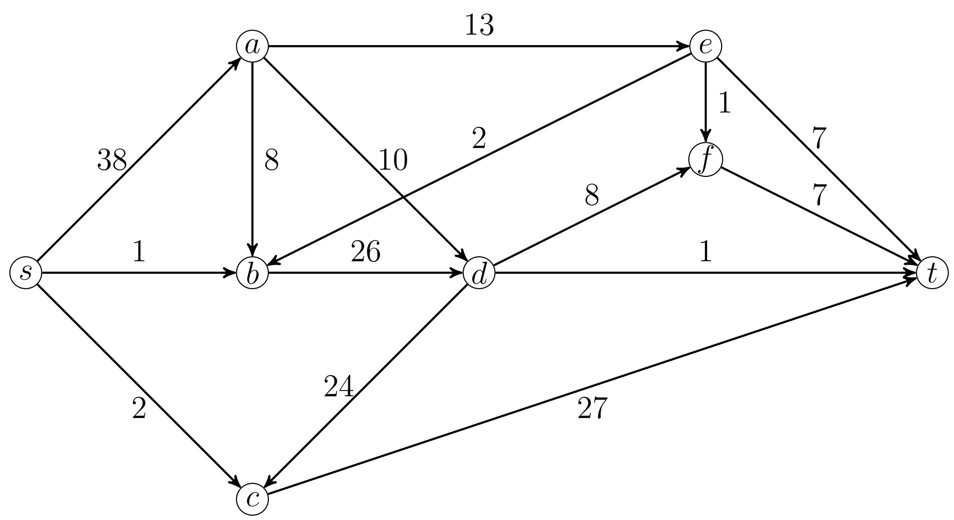
   (a) Give an example of a graph G on n vertices with a maximum number of edges such that $\chi(G) \leq 4$.

   (b) Give an example of a graph G such that $\chi(G) = 5$ but $\omega(G) \leq 4$. (Recall $\omega(G)$ is the clique number of G.)

   (c) Give an example of a k-connected graph that is not Hamiltonian.

6. (10 points) Assume G is a graph that is twice-color-critical. That is, for every pair of distinct vertices x and y, $\chi(G - x - y) = \chi(G) - 2$. Prove that G must be the complete graph.

7. (10 points) In the network below, find a maximum flow from s to t and prove that your answer is correct.



8. (10 points) Use Dijkstra's Algorithm to find the shortest path from v to w in the graph G below.