MATH 663
Spring 2010
In-Class Midterm

Books and notes are not allowed. There are seven problems each worth 10 points. You have one hour to complete the exam.

1. (10 points) Determine whether or not the sequence $7, 4, 3, 3, 2, 2, 2, 2, 1, 1, 1$ is graphic. (Show your work. Explain your answer.)

2. (10 points) In the matrix below, $a_{i,j}$ gives the weight of the directed edge from x_i to x_j for a digraph D. Use Dijkstra's Algorithm to find $d(x_1, x_i)$ for $i = 2, 3, 4, 5$.

$$\begin{pmatrix} 0 & 5 & \infty & 40 & 30 \\ 8 & 0 & 20 & \infty & 15 \\ 30 & 40 & 0 & 17 & 8 \\ 25 & 15 & 15 & 0 & \infty \\ 20 & 16 & 10 & 10 & 0 \end{pmatrix}$$

3. (a) (2 points) Define *strong component* in a digraph.

   (b) (8 points) Prove that the strong components of a digraph are pairwise disjoint.

4. (a) (5 points) Find a tree with Prüfer Code 4258268.

   (b) (5 points) How many trees on n vertices (with vertex labels $1, 2, \cdots, n$) have vertex n as a leaf?

5. (a) (2 points) Give the definition of the *diameter* and the *radius* of a graph.

(b) (3 points) Give an example of a simple graph G on n vertices for $n \geq 3$ such that diam(G)=rad(G)=2.

(c) (5 points) Prove that if $diam(G) = d$ then $\alpha(G) \geq \lceil d/2 \rceil$. (Recall that $\alpha(G)$ is the size of the largest independent set of vertices in G.)

6. (a) (4 points) State the König-Egerváry Theorem.

(b) (6 points) Use the König-Egerváry Theorem to prove that every bipartite graph G has a matching of size at least $e(G)/\Delta(G)$. (Recall $e(G)$ is the number of edges and $\Delta(G)$ is the maximum degree of G.)

7. (a) (2 points) What is an *even cycle*?

(b) (8 points) Let G be a simple graph such that $\delta(G) \geq 3$. Prove that G contains an even cycle.