Disclaimers: If a definition, term, or notation was discussed in class and/or appeared on the homework, you are expected to know it. There is no claim that this review is perfect.

The Final Exam for Fairbanks students will be Friday December 15, 10:15-12:15.

In length and style, the Final Exam will be just like the midterms. Unlike the Midterms, the problems will be ordered by difficulty from straightforward to more challenging.

Section 1.2 Degree of a Vertex

- terminology/notation: degree, average degree, degree per vertex,
- results to know: Prop 1.2.1 (Sum of degrees is even.)
- results to know how to prove: Prop 1.2.1 (Sum of degrees is even.)

Section 1.3 Paths and Cycles

- terminology/notation: path, cycle, independent path, AB-path, distance, girth, central vertex, radius, diameter.
- results to know: Prop 1.3.1 ($\delta(G) geq 2$ implies G contains a cycle of length at least δ or path of length at least $\delta+1$.
- results to know how to prove: Prop 1.3.2 (If G has a cycle, then $g(G) \leq 2\text{diam}(G)+1$, Prop 1.3.3 ($\text{rad}(G) \leq k$ and $d = \Delta(G)$ implies $\| G \| \leq \frac{d}{d-2}(d-1)^k$.)

Section 1.4 Connectivity

- terminology/notation: connected, component, separator, separating set of vertices, separating set of edges, cut vertex, bridge, k-connected, connectivity k, k-edge-connected, edge-connectivity k
- results to know: Prop 1.4.1 (In a connected graph, vertices can be ordered such that induced graphs are connected.) , Prop 1.4.2. ($\kappa(G) \leq \lambda(G) \leq \delta(G)$.)

Section 1.5 Trees and Forests

- terminology/notation: tree, forest, acyclic, leaf,
- results to know: Thm 1.5.1 (equivalent formulation of a tree), Cor 1.5.3 (# edges in a tree)
- results to know how to prove: Thm 1.5.1, Cor 1.5.3

Section 1.6 Bipartite Graphs

- terminology/notation: r-partite graph, bipartite graph, vertex class, complete r-partite graph
- results to know: Proposition 1.6.1 (characterization of bipartite graphs)

Section 1.7 Contraction and Minors

- terminology/notation: subdivision, topological minor, (regular) minor

Section 1.8 Euler Tours

- terminology/notation: Euler tour, Eulerian
- results to know by name: Thm 1.8.1 Euler's Theorem
- results to know how to prove: Thm 1.8.1 Euler's Theorem

Section 2.1 Matching in Bipartite Graphs

- terminology/notation: matching, factor, vertex cover, alternating path, augmenting path, stable matching, Hall's condition
- results to know by name: Thm 2.1.1 König's Theorem, Thm 2.1.2 Hall's Theorem,
- results to know: Cor 2.1.3 (k-regular and bipartite implies the existence of a 1-factor.)
- results to know how to prove: Thm 2.1.2 Hall's Theorem, Cor 2.1.3

Section 2.2 Matching in General Graphs

- terminology/notation: Tutte's condition
- results to know by name: Thm 2.2.1 Tutte's Theorem
- results to know: Cor 2.2.2 (Every bridgeless cubic graph contains a 1-factor)
- results to know how to prove: Cor 2.2.2

Section 3.1 2-Connected Graphs and Subgraphs

- terminology/notation: H-path, block, block graph
- results to know: Prop 3.1.1 (H-path construction of 2-connected graphs.) , Lemma 3.1.2, Lemma 3.1.3 (These are descriptions of the block structure of graphs), Lemma 3.1.4 (The block graph of a graph is a tree.)
- results to know how to prove: Lemma 3.1.4

Section 3.3

- terminology/notation: disjoint A-B paths, independent ab-paths
- results to know by name: Thm 3.3.1 Menger's Theorem, Thm 3.3.6 Global Menger's Theorem
- results to know and to know how to prove: Cor 3.3.4, Cor 3.3.5, Thm 3.3.6 Global Menger's Theorem

Chapter 4: Planar Graphs

- terms: plane graph, face, outer face, outer planar, maximally planar, plane triangulation, maximal plane graph.
- theorems to remember:
  - Thm 4.4.1 Jordan Curve theorem
  - Prop 4.2.4: A plane forest has exactly one face.
  - Prop 4.2.6: In a 2-connected plane graph, every face is bounded by a cycle.
  - Prop 4.2.8 A plane graph on at least three vertices is maximally plane if and only if it is a plane triangulation.
  - Cor 4.2.10 A plane graph has at most $3n-6$ edges (provided $n \geq 3$). Every plane triangulation with n vertices has exactly $3n-6$ edges.
  - Cor 4.2.11 A plane graph contains neither a K^5 nor a $K_{3,3}$ as a subgraph.
  - Prop 4.4.1 Every maximal plane graph is maximally planar. For a planar graph, maximally planar is equivalent to having $3n-6$ edges (provided $n \geq 2$).
- theorems to know by name: Thm 4.2.9 Euler's Formula, Thm 4.4.6 Kuratowski's Theorem A graph is planar if and only if it has no K^5 or $K_{3,3}$ minor.

Chapter 5: Coloring

- terms: coloring, vertex coloring, edge coloring, k-coloring, k-edge-coloring, k colorable, k-edge colorable, k-chromatic, k-edge-chromatic, chromatic number, edge chromatic number, $\chi(G)$, $\chi'(g)$, greedy coloring, Mycielski's construction
- theorems to remember:
  - Lemma 5.2.3 Every k-chromatic graph contains a subgraph of minimum degree at least $k-1$.
  - Prop 5.3.1 If G is bipartite, then $\chi'(G) = \Delta(G)$.
- theorems to know by name:
  - Them 5.2.4 Brook's Theorem Let G be a connected graph. Then $\chi(G) \leq \Delta(G)$ or G is a complete graph or G is an odd cycle.
  - Thm 5.3.2 Vizing's Theorem For every (simple) graph G, $\Delta(G) \leq \chi'(G) \leq \Delta(G)+1$.

Chapter 6: Flows

- terms: network, capacity, flow, integral flow $\overrightarrow{E}(G)$, cut, $\overrightarrow{E}(X,Y)$ $\overrightarrow{e}$, $\overleftarrow{e}$, $c(X,Y)$, $f(X,Y)$, value of a flow, $|f|$, capacity of a cut.
- theorems to remember:
  - Prop 6.2.1: In a network N with cut S, $f(S,\overline{S}) = f(s,V)$.

- theorems to know by name: Thm 6.2.2 Ford Fulkerson In every network, the maximum value of a flow is equal to the minimum capacity of a cut.

Chapter 7: Extremal Graph Theory

- terms: Turán graph, extremal graph, extremal number, $ex(n,H)$
- theorems to know by name: Thm 7.1.1 Turán For all integers r and n with $r > 1$, if G is K^r-free and $|E(G)| = ex(n,K^r)$, then $G \cong T^{r-1}(n)$.

Chapter 8: Infinite Graphs No questions here.

Chapter 9: Ramsey Theory

- Know Theorem 9.1.1 For every $r \in \mathbb{N}$ there exists an $n \in \mathbb{N}$ such that every graph on at least n vertices either contains a K^r or its complement contains a K^r.
- Know the meaning of $R(G,H)$ and how to make arguments that $R(G,H) = n$.

Chapter 10: Hamilton Cycles

- terminology: hamilton cycles and path, hamiltonian graphs
- Be able to prove Dirac's Theorem (Thm 10.1.1) If G is an n-vertex graph (where $n \geq 3$) and $\delta(G) \geq n/2$, then G is hamiltonian.
- Know Prop 10.1.2: If G is an n-vertex graph (where $n \geq 3$) and $\alpha(G) \leq \kappa(G)$, then G is hamiltonian.
- Know how to makes sense of and use Chvatal's degree criteria for hamiltonicity (Theoerem 10.2.1) The sequence $(a_1, a_2, \cdots, a_n)$ is hamiltonian iff for every $i < n/2$, $a_i \leq i \Longrightarrow a_{n-i} \geq n - i$.