Spring 2025 **Math 316**

Exam 1

Name: _____

Rules:

- Partial credit will be awarded, but you must show your work.
- No notes, books, or cell phones are allowed.
- Calculators are allowed.

NOTE: The exam is formatted in order to provide plenty of space. A complete answer does **not** need to fill the given space. Indeed, it is not necessary.

Problem	Possible	Score
1	15	
2	15	
3	20	
4	20	
5	15	
6	15	
Extra Credit	(5)	
Total	100	

1. (15 points) Use ancient Egyptian methods to complete the following calculations.

   (a) Multiply 50 by 42.

   (b) Divide 159 by 42.

   (c) Show that unit fraction decomposition is not unique by giving an example of a rational number q and two different unit fraction decompositions of q (where the unit fractions must be distinct).

2. (15 points) This problem concerns the base-60 numerical representation of the ancient Babylonians. We will use our textbook's notation.

   (a) Write the base 60 number $3;45$ in base 10.

   (b) Explain, with computation, how $3;45$ can be the reciprocal of 16.

   (c) Describe two advantages of the Babylonian base 60 representation of numbers when compared to ancient Egyptian representation.

3. (20 points) This question is about the idea of **incommensurable quantities.**

   (a) What did ancient Greek mathematicians mean by the words "line segments AB and BC are incommensurable"?

   (b) How, when, and who discovered the existence of incommensurable line segments.

   (c) Describe (with pictures) how ancient Greek mathematicians could have constructed a line segment of length $\sqrt{n}$ for any positive integer n.

   (d) Why is the existence of incommensurable quantities a crisis for Greek mathematicians?

4. (20 points) This question is about Euclid's **Elements**.

   (a) When and where was it written?

   (b) Describe its contents including its structure and its style of exposition.

   (c) State the 5th Postulate and explain how it is different from the other four postulates.

   (d) Compare Euclid's proof of the infinitude of primes from a modern one.

   (e) State two notable results found in Euclid's **Elements** (other than the proposition that there are an infinite number of primes.)

5. (15 points) This question is concerns the notion of the quadrature of geometric figures.

   (a) Explain the what is meant by the statement that Hippocrates accomplished the quadrature of a lune and explain why this was important. (You may want to draw pictures to illustrate your words.)

   (b) Describe the method Archimedes used in his quadrature of a parabolic segment. (You may want to draw pictures to illustrate your words.)

6. (15 points) Short answer.

   (a) Explain how Archimedes' statement that **the area of a circle is to the square on its diameter as 11 is to 14** can be interpreted as an estimation of π.

   (b) Describe two ways in which Diophantus' notation in **Arithmetica** was novel and describe two of its limitations.

   (c) Give one example of cut-and-paste algebra. A complete answer includes a picture (or pictures) and a modern algebraic identity that the picture (or pictures) demonstrates.

Extra Credit: (5 points) Below is Problem 27 from Book I from Diophantus' **Arithemetica** as it appears in Thomas Heath's 1910 translation. Provide a modern algebraic explanation of the problem and the solution. Full points for explaining the trick Diophantus is employing that make his scheme work.

To find two numbers such that their sum and product are given numbers.

Given sum 20, given product 96.
2x the difference of the required numbers.
Therefore the numbers are $10 + x$, $10 - x$.
Hence, $100 - x^2 = 96$.
Therefore, $x = 2$, and
the required numbers are 12, 8.