Mon 20 March

Reminders
- Wed is review
- Review sheet posted
- First problem posted
- Midterm 2 is Friday

on public course webpage

Today
- Quick Review of Fri's discussion
- Finish 5.5

Quick 5.5 Summary (thus far)

- Al-Khwarizmi (780-850, Baghdad)
  - Solved all 6 quadratic equations
  - Positive coefficients only
  - Numerical solutions
  - Cut-and-paste geometric justifications of both solutions and algebraic operations
  - Rhetorical explanations with integers being the only symbols
  - Only recognized positive solutions
  - For "Squares and Numbers Equal to Roots", he gave two solutions:

$ax^2 + c = bx$ has solutions

$$x = \frac{b}{2} \pm \sqrt{\left(\frac{b}{2}\right)^2 - c}$$

  - He says "If addition does not give your answer, then subtraction will."

Ex $x^2 + 21 = 10x$

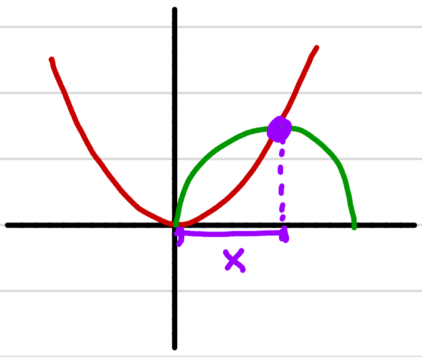
$x^2 - 10x + 21 = 0$

$(x-7)(x-3) = 0$

$x = 3, x = 7$

Khayyam (1048-1123, Persian, worked various places)

- Solved all 19 cubic equations
- Solutions were geometric (line segments resulting from intersections of conic sections)
- Ex



- Used properties of conics we no longer use.

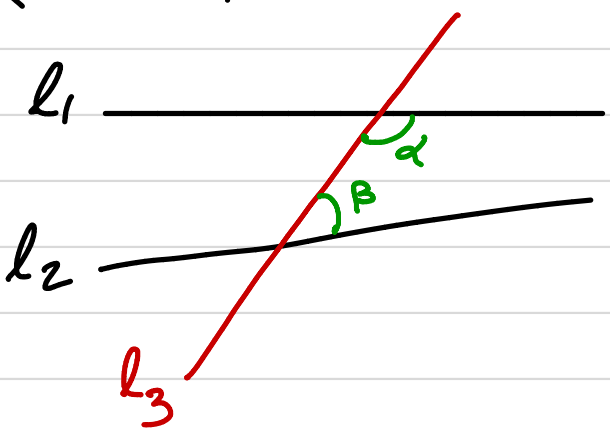
Modern Parabola
$y = kx^2$

Khayyam's Parabola
The set of points equidistant from a point P and line ℓ.

- Like al Khwarizmi,
  - rhetorical
  - geometric proof/justification
  - positive coefficients
  - only recognized positive solutions
  - instances of multiple positive solutions

Euclid's 5^{th} Axiom

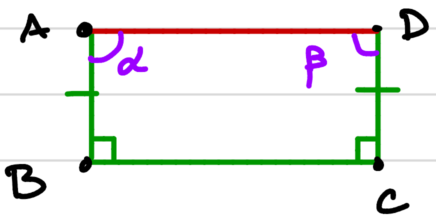
- (Euclid, <u>Elements</u>)



ℓ_1, ℓ_2, ℓ_3 straight lines

If $\alpha + \beta < 180°$, then ℓ_1 and ℓ_2 intersect on the right.

- Thabit ibn Qurra (836-901, Turkey, Baghdad)
  Goal: Prove the 5^{th} axiom from Axioms 1-4 and Propositions 1-28
  Strategy: <u>Saccheri</u> Quadritateral
  ↳ Giovanni Saccheri (1667-1733)



If $\angle B = \angle C = 90°$ and $AB = CD$, then $\alpha = \beta = 90°$.

Argument: If $\alpha < 90°$, then $\overline{CD} < \overline{AB}$. $\Rightarrow\Leftarrow$
If $\alpha > 90°$, then $\overline{CD} > \overline{AB}$. $\Rightarrow\Leftarrow$

- Khayyam (1048-1123) replaced Euclid's description w/ "Converging lines intersect."
  (Idea: A 5^{th} axiom is needed, but Euclid's can be replaced w/ axiom that is simpler + more intuitive.)

- Nasir al-Din al-Tusi (1201-1274, Baghdad)

Addition Comments about Mathematical Contributions of Islamic Mathematicans during roughly 700-1500

- Use of + manipulation of square roots
  answers like: $\sqrt{a-\sqrt{b}+\sqrt{c}} - \sqrt{d}$

- Generalized Pythagorean Theorem.
  (ie a version that applies to non-right triangles)

- Basic combinatorial formulas
  eg: • the coefficient of a^3b^2 in $(a+b)^5$ is $\binom{5}{3}$
  • $\binom{n}{k} = \binom{n-1}{k}\binom{n}{k-1}$

- Sums of Series
  • $1^2+2^2+\ldots+n^2 = \frac{(2n+1)(n+1)(n)}{6}$

- Astronomy
  • trigonometry (sine, cosine, tangent)
  • skill and endurance w/ numerical calculation

- Both practical and valued proof

Chinese Mathematics (roughly 300 bc - 1600 ad)

- Texts
  - (300 bc) Arithmetic Classic of the Gnomon and the Circular Paths of Heaven
  - (200 bc?; 263 ad Liu Hui commentary) Nine Chapters on the Mathematical Art
    - text book, audience is students
    - practical
    - consolidation of known mathematics
    - Topics
      - areas + volumes
      - sums of arithmetic progressions
      - right triangle properties/calculations
      - proportions
      - calculate square and cube roots
  - (263 ad Liu Hui) Sea Island Mathematical Manual
    - Determine distance and height of an island from shore

    - $27.30 on Amazon (paperback)
  - (1247, Ch'in Chu-shao) Mathematical Treatise in Nine Sections
    - ink color to indicate negative numbers
    - Use of "o" for zero
    - polynomials of degree 4.

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- (1248, 1259, Li Ye) Sea Mirror Measurements, Old Mathematics in Expanded Sections
  - represented negative numbers with strike (essentially -3 is $\cancel{3}$)
  - solutions to quadratic equations

- (1299, 1303, Chu Shih-chieh)

  Introduction to Mathematical Studies
  Precious Mirror of the Four Elements
  - Pascal's Triangle (coeff. of binomial expansion)
  - higher degree polynomials
  - sums of sequences

Other Developments

- approximations of π via inscribed polygons a la Archimedes
- Technique of Gaussian elimination to solve systems of linear equations including the use of negative coefficients in middle computations

Solve

$$\begin{aligned} x + y + 3z &= 12 \\ x + 2y + z &= 7 \\ 2x + 2y + 8z &= 30 \end{aligned}$$

subtract eq 1 from eq 2, and 2 eq 1 from eq 3.

$$\begin{aligned} x + y + 3z &= 12 \\ y - 2z &= -5 \\ 2z &= 6 \end{aligned}$$

$z = 3$, $y = 1$, $x = 2$

- European mathematics introduced to Chinese mathematicias ≈ 1600.