Section 4.3

- What is our modern definition of a prime number and what is Euclid's? (To see Euclid's definition go here)
- What does Euclid mean by a number in definition 2?
- Give an example of two relatively prime numbers and two relatively composite numbers.
- Find the prime factorization of 128 and 24.
- Find the greatest common divisor of 128 and 24.
- Find the least common multiple of 128 and 24.
- What is the Fundamental Theorem of Arithmetic?
- You get a gold star if you recognize the modern terminology for the first two propositions in Book VII.

Definition 1
A *unit* is that by virtue of which each of the things that exist is called one.

Definition 2
A *number* is a multitude composed of units.

Definition 3
A number is *a part* of a number, the less of the greater, when it measures the greater;

Definition 4
But *parts* when it does not measure it.

Definition 5
The greater number is a *multiple* of the less when it is measured by the less.

Definition 6
An *even number* is that which is divisible into two equal parts.

Definition 7
An *odd number* is that which is not divisible into two equal parts, or that which differs by a unit from an even number.

Definition 8
An *even-times-even number* is that which is measured by an even number according to an even number.

Definition 9
An *even-times-odd number* is that which is measured by an even number according to an odd number.

Definition 10
An *odd-times-odd number* is that which is measured by an odd number according to an odd number.

Definition 11
A *prime number* is that which is measured by a unit alone.

Definition 12
Numbers *relatively prime* are those which are measured by a unit alone as a common measure.

Definition 13

A *composite number* is that which is measured by some number.

Definition 14

Numbers *relatively composite* are those which are measured by some number as a common measure.

Definition 15

A number is said to *multiply* a number when the latter is added as many times as there are units in the former.

Definition 16

And, when two numbers having multiplied one another make some number, the number so produced be called *plane,* and its *sides* are the numbers which have multiplied one another.

Definition 17

And, when three numbers having multiplied one another make some number, the number so produced be called *solid,* and its *sides* are the numbers which have multiplied one another.

Definition 18

A *square number* is equal multiplied by equal, or a number which is contained by two equal numbers.

Definition 19

And a *cube* is equal multiplied by equal and again by equal, or a number which is contained by three equal numbers.

Definition 20

Numbers are *proportional* when the first is the same multiple, or the same part, or the same parts, of the second that the third is of the fourth.

Definition 21

Similar plane and *solid* numbers are those which have their sides proportional.

Definition 22

A *perfect number* is that which is equal to the sum its own parts.

Use the Euclidean Algorithm to find the greatest common divisor of 1260 and 138.

[Note: $1260 = 2^2 \cdot 3^2 \cdot 5 \cdot 7$ and $138 = 2 \cdot 3 \cdot 23$]

We will mention 6 results

Book VII

Proposition 1

When two unequal numbers are set out, and the less is continually subtracted in turn from the greater, if the number which is left never measures the one before it until a unit is left, then the original numbers are relatively prime.

Proposition 2

To find the greatest common measure of two given numbers not relatively prime.

Book IX

Proposition 14.

If a number is the least that is measured by prime numbers, then it is not measured by any other prime number except those originally measuring it.

Proposition 20.

Prime numbers are more than any assigned multitude of prime numbers.

Proposition 21.

If as many even numbers as we please are added together, then the sum is even.

Proposition 35.

If as many numbers as we please are in continued proportion, and there is subtracted from the second and the last numbers equal to the first, then the excess of the second is to the first as the excess of the last is to the sum of all those before it.

History of the Book itself

- Written 300's bc

- Theon of Alexandria 335-405 ce wrote a version (comments and additions) in Greek

- Many translations of Theon's version into Arabic by various Islamic scholars

- Arabic versions translated into Latin by European scholars.

- In 1808, a scholar found a manuscript of

  Elements w/o an addition known to be due to Theon of Alexandria.

The Value of a Book like Elements

- Researchers talk to each other

- Books like Elements talk to students

- Requires the author to have mastery of

  and perspective on the subject.

- A good textbook can radically change accessibitity.

- Ex: Theon of Alexandria

  Emilie du Châtelet , 1706-1749, France
  Maria Agnesi , 1718-1799, Italy

  (Newton's Principia 1687)

  (Leibniz's Calc appears 1684-1693)