MATH 306: Introduction to the History and Philosophy of Mathematics
Spring 2013
Thursday May 9, 2013
Final Exam — Part I

NAME:______________________________

This part contains 1 question worth 24 points. It is closed-book, closed note. You should spend no more than 20 minutes on this section. When you have finished, turn in Part I and you will be given Part II. You have a total of 2 hours for Part I and Part II together.

1. (3 points each) For each mathematician listed below, state about when they lived, about where they lived, and describe their most significant mathematical contributions (at least two).

   (a) Archimedes (when: ____________) (where: ________________________)

   (b) 'Umar Al-Khāyammī (when: ____________) (where: ________________________)

   (c) Muhammad Al-Khwārizmī (when: ____________) (where: ________________________)

   (d) Rene Descartes (when: ____________) (where: ________________________)

(e) Euclid (when: ____________) (where: ________________________)

(f) Pierre Fermat (when: ____________) (where: ________________________)

(g) Gottfried Leibniz (when: ____________) (where: ________________________)

(h) Isaac Newton (when: ____________) (where: ________________________)

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Final Exam — Part II

NAME:______________________________

This part contains 10 questions worth 76 points plus and extra credit problem worth 5 points.

1. (10 points)

   (a) Use the method of false position to solve the following problem:

   Johnny has a bag of cookies and gives half of them to friend A. Then he gives a third of the remaining cookies to friend B. Finally, after giving a fourth of what he has left to friend C, Johnny finds he has two and one-fourth cookie left. How many cookies did Johnny have in his bag at the beginning?

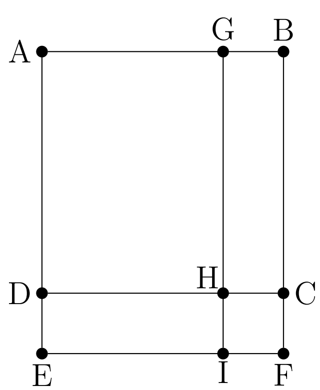
   (b) Explain why the modification in italics changes the problem such that a routine application of false position no longer works:

   Johnny has a bag of cookies and gives half of them to friend A. Then he gives a third of the remaining cookies to friend B. *Friend A then returns 2 of her cookies saying she's afraid she will eat them all at once and get really sick.* Finally, after giving a fourth of what he now has left to friend C, Johnny finds he has three cookies left. How many cookies did Johnny have in his bag at the beginning?

2. (6 points) Explain what the ancient Greek mathematicians meant when they claimed that the side of a square was incommensurable with its diagonal. Don't use modern terminology. Describe how the Greeks were thinking of it.

3. (6 points) Describe at least three ways in which the development of hyperbolic geometry was different from that of projective geometry.

4. (8 points) Ancient Babylonians knew the algebraic fact that $(u+v)(u-v) = u^2 - v^2$. Explain how the picture below illustrates this algebraic fact. Identify any implicit assumptions and why these are reasonable in this context.



5. (6 points) Find the solutions to $x^3 - 5x^2 + 5x - 1 = 0$ using Albert Girard's method of factions and the observation that $x = 1$ is clearly a solution.

6. (6 points) A modern precalculus text introduces the logarithm as the inverse of an exponential function. Explain how the origins of the logarithm are very different from this modern view.

7. (6 points) Use Abraham bar Hiyya's table on page 195 to find the length of a chord that cuts off an arc of length 18 in a circle of radius 10.

8. (10 points)

   (a) (3 points) Give two examples of how the coordinate geometry of Descartes and Fermat was rather different than our modern version.

   (b) (3 points) Describe one fundamental way in which Descartes' approach to coordinate geometry and Fermat's approach to coordinate geometry were quite different from each other.

   (c) (2 points) Give an example of a mathematical idea crucial to the development of coordinate geometry and explain about when it developed.

   (d) (2 points) Give an example of a mathematical idea for which the existence of coordinate geometry was crucial to its development and explain about when this development occurred.

9. (10 points)

   (a) Identify first examples of (i) a base 10 numerical system, (ii) a base 10 *positional* system and (iii) a base 10 positional system with decimal fractions. (You should state the *when* and the *who*.)

   (b) Estimate the length of time between the introduction of a base 10 positional system in Europe and the use of decimal fractions in Europe and provide some explanantion of this delay.

10. (8 points)

   (a) Describe how 'Umar Al-Khāyammī's solution and Girolamo Cardano's solution to the cubic were different.

   (b) Both mathematicians knew their solutions were in some fundamental ways distinctly unsatisfactory. Explain how.

11. (5 points) EXTRA CREDIT
Several years before James Garfield became President of the United States, he found a new proof of the Pythagorean Theorem. It was published in 1876 in the *New England Journal of Education*. Starting with a right triangle ABC, extend AC and construct a congruent triangle EAD as indicated in the figure below. (Thus, CD forms a straight line by construction.) Now draw line segment EB to produce quadrilateral EBCD. Prove that $a^2 + b^2 = c^2$ by *relating the area of the quadrilateral to the area of the three triangles.*