

**Midterm — Solutions to essay-style questions**

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Compare the mathematics of Euclid, Archimedes, and Diophantus. You should have included a substantial subset of the following ideas.

- Both Archimedes and Diophantus were writing about new mathematics that they discovered/developed themselves which is in contrast to Euclid whose primary contribution was the organization and logical scaffolding of known mathematics.
- Another way in which Euclid's Elements is profoundly different is in the complete absence of numbers. Both Archimedes and Diophantus were, by comparison, rather numerical.
- Euclid and Archimedes insisted upon providing rigorous proofs. Diophantus did not. This was partly because his solutions were on the surface numerical and thus the proof was obvious. However, if you believe that his numerical solutions were really an attempt to describe a more general algorithm, Diophantus was handicapped by insufficient notation.
- Archimedes is the only one of the three to attack obviously applied problems. The problems and the formulation of solutions of Diophantus and Euclid are very strictly pure as the driven snow.
- Archimedes was the only one of the three to provide any motivation to the problems he chose to attack or his method of solution. The order of the theorems and the strategies of the proofs of Euclid are utterly without explanation as are the algebraic tricks of Diophantus.
- They all three worked on different sorts of problems. Euclid feels largely geometric though one should not forget the elementary number theory and theory of incommensurables. Archimedes built upon the work of Euclid, particularly geometrically such as ratios of various geometric objects, but also considered numerical issues such as a numerical approximation of  $\pi$ . The schemes for solving indeterminate equations and the proto-algebraic notation developed by Diophantus seem utterly unlike anything Euclid or Archimedes did.

Ancient Babylonians, Brahmagupta, and Al-Khwarizmi all solved quadratic equations. Compare their different approaches including both ways in which they are similar and ways in which they are different. Again, your answer should have included a substantial subset of the following observations.

- All three considered only positive solutions.
- All three come from an intellectual history of cut-and-paste algebra, though Brahmagupta's more general formula seems to have evolved beyond this insistence of concreteness.
- Brahmagupta was the only one to allow negative coefficients. Both ancient Babylonians and Al Khwarizmi required strictly positive coefficients. As a result, Brahmagupta was the only one who could even attempt a general formula. The other two were required to have different formulae in the case of Al Khwarizmi or processes in the case of the Babylonians for different quadratic equations.
- The ancient Babylonians did not have the rhetorical algebra of Brahmagupta or Al Khwarizmi. They also were not systematic and comprehensive as both Brahmagupta and Al Khwarizmi.

- Al Khwarizmi was noticeably different from the other two in that he clearly identified conditions under which two (positive) solutions could be found and he insisted upon rigorous geometric proof of his assertions. He shows the influence of the ancient Greek traditions.
- Al Khwarizmi also appears influenced by and to build upon the work of the ancient Babylonians while the work of Brahmagupta is independent of the other two