Matrix Factorization Example

Set up a 3x3 matrix

```
% Define the initial 3x3 matrix
A = [2 1 1;
     4 -6 0;
     -2 7 2]
```

```
A = 3×3
     2     1     1
     4    -6     0
    -2     7     2
```

First: A=LU

Elimination Matrices

```
E21 = [1 0 0;-2 1 0; 0 0 1];
E31 = [1 0 0; 0 1 0; 1 0 1];
% Need to apply these to A to know the next steps:
E31*E21*A
```

```
ans = 3×3
     2     1     1
     0    -8    -2
     0     8     3
```

```
E32 = [ 1 0 0; 0 1 0; 0 1 1];
U = E32 * (E31 * (E21 * A))
```

```
U = 3×3
     2     1     1
     0    -8    -2
     0     0     1
```

```
% Construct L from thin air
% Place multipliers in their positions
L = [1 0 0; 2 1 0; -1 -1 1]
```

```
L = 3×3
     1     0     0
     2     1     0
    -1    -1     1
```

```
% Check that A=LU
checkLU = A - L * U;
disp(checkLU);
```

```
     0     0     0
     0     0     0
     0     0     0
```

Where did this come from?

```
% In fact, L "undoes" elimination:
E32inv = [ 1 0 0; 0 1 0; 0 -1 1];
E31inv = [1 0 0; 0 1 0; -1 0 1];
E21inv = [1 0 0; 2 1 0; 0 0 1];
alt_L = E21inv * E31inv * E32inv
```

```
alt_L = 3×3
     1     0     0
     2     1     0
    -1    -1     1
```

```
% How do we know this is always possible?
% How do we know L is always lower triangular?
```

Second: A=LDU

```
%% D consists of the
%% diagonal entries
D=[2 0 0; 0 -8 0; 0 0 1];

% Construct the new U
% from the old U by
% dividing by the diagonal
% entries

U2 = [1 1/2 1/2;
    0 1 1/4;
    0 0 1];

% Verify the decomposition by reconstructing A from L, U, and D
A_reconstructed = L * D * U2
```

```
A_reconstructed = 3×3
     2     1     1
     4    -6     0
    -2     7     2
```

```
check = A - A_reconstructed
```

```
check = 3×3
     0     0     0
     0     0     0
     0     0     0
```