This quiz has two problems worth 10 points.

1. (3 points) Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 3 \\ 3 & 0 & 8 \end{bmatrix}$ and $\mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$. Write the matrix · vector product $A\mathbf{x}$ as:

(a) a linear combination of the columns of A

$$x\begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + z\begin{bmatrix} 2 \\ 3 \\ 8 \end{bmatrix}$$

(b) a single vector

$$\begin{bmatrix} x - y + 2z \\ y + 3z \\ 3x + 8z \end{bmatrix}$$

(c) as dot products of rows of A

$$\begin{bmatrix} \text{row}1 \cdot \vec{x} \\ \text{row}2 \cdot \vec{x} \\ \text{row}3 \cdot \vec{x} \end{bmatrix} = \begin{bmatrix} (1,-1,2) \cdot (x,y,z) \\ (0,1,3) \cdot (x,y,z) \\ (3,0,8) \cdot (x,y,z) \end{bmatrix}$$

2. (7 points) Let $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} -4 \\ 1 \\ -2 \end{bmatrix}$

(a) Is the angle between **v** and **w** acute, right, or obtuse? Explain your answer. (Note you were not asked to find the angle.)

$\vec{v} \cdot \vec{w} = 1(-4) + 2(1) + 2(-2) = -4 + 2 - 4 = -6$

Since $-6 < 0$, the angle is obtuse.

(b) Find a **nonzero** vector that is perpendicular to **v** and show that you are correct.

Pick $\vec{a} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$. $\vec{a} \cdot \vec{v} = 0$

(c) Find a **unit** vector **u** in the same direction as vector **v**.

$\|\vec{v}\| = \sqrt{1^2 + 2^2 + 2^2} = 3$

$\vec{u} = \frac{1}{\|\vec{v}\|}\vec{v} = \frac{1}{3}(1,2,2) = \left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$

(d) Find a vector **a** such that the set of vectors **v**, **w**, and **a** are **dependent**.

Choose $\vec{a} = \vec{v} + \vec{w} = \begin{bmatrix} -3 \\ 3 \\ 0 \end{bmatrix}$