Linear Algebra Quiz 3 Name: Solutions

This quiz has two problems worth 10 points.

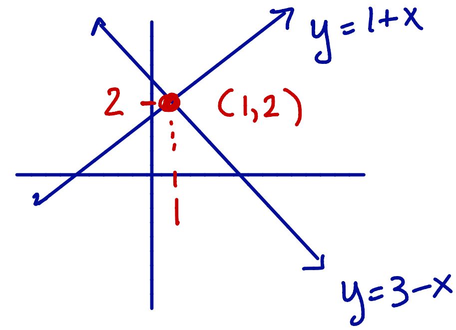
1. (5 points) The question below concerns the system of equations $x + y = 3$ and $-x + y = 1$.

   (a) (1 point) Write this system as a matrix·vector equation of the form $A\mathbf{x} = \mathbf{b}$.

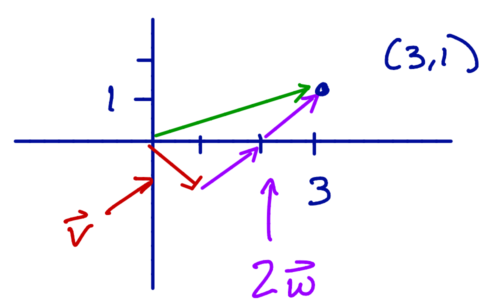
$$\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

   (b) (4 points) This system of equations has solution $x = 1$, $y = 2$. **Draw** the row and column pictures in this case. Label which one is which. Your pictures don't need to be perfect but they should be roughly correct and labelled.

row : $\left.\begin{array}{r} x+y=3 \\ -x+y=1 \end{array}\right]$ or $\left[\begin{array}{l} y=3-x \\ y=1+x \end{array}\right.$



Column $\quad 1 \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix} + 2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ or $1\,\vec{v} + 2\vec{w} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$



2. (5 points) Reduce the system below to upper triangular form using two row operations. List the pivots and the multipliers. Solve by back substitution.

$$\begin{array}{ccccc} -2 & & & & -2 \quad -0 \\ x & & & + & z = 0 \\ 2x & + & 3y & - & z = 3 \\ & & 4y & + & z = 2 \end{array} \xrightarrow{r_2 - 2r_1} \begin{array}{l} x \qquad + z = 0 \\ 3y - 3z = 3 \\ 4y + z = 2 \\ -4 \quad +4 \quad -4 \end{array} \xrightarrow{r_3 - \left(\frac{4}{3}\right) r_2} \begin{array}{l} x \qquad + z = 0 \\ 3y - 3z = 3 \\ \qquad 5z = -2 \end{array}$$

multipliers: $\ell_{21} = 2$

$\ell_{32} = \frac{4}{3}$

Pivots: 1, 3, 5

Back Substitution:

$z = -2/5$

$y = 1 + z = \frac{3}{5}$

$x = \frac{2}{5}$

$$\vec{x} = \begin{bmatrix} 2/5 \\ 3/5 \\ -2/5 \end{bmatrix}$$