This quiz has three problems worth 10 points.

1. The questions below concern the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 1 & 0 & 4 \end{bmatrix}$.

   (a) (4 points) Use the Gauss-Jordan method to calculate A^{-1}. (That is, you are going to use the augmented matrix $[A \quad I]$. There are not a lot of steps in this case!)

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 1 & 0 \\ 1 & 0 & 4 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\ell_{31}=1} \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 1 & 0 \\ 0 & 0 & 2 & -1 & 0 & 1 \end{array}\right] \xrightarrow[\ell_{23}=\frac{1}{2}]{\ell_{13}=1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 0 & -1 \\ 0 & 2 & 0 & \frac{1}{2} & 1 & -\frac{1}{2} \\ 0 & 0 & 2 & -1 & 0 & 1 \end{array}\right]$$

$1 \quad -\frac{1}{2} \quad 0 \quad \frac{1}{2}$

$$\xrightarrow[r_3 \cdot \frac{1}{2}]{r_2 \cdot \frac{1}{2}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 0 & -1 \\ 0 & 1 & 0 & \frac{1}{4} & \frac{1}{2} & -\frac{1}{4} \\ 0 & 0 & 1 & -\frac{1}{2} & 0 & \frac{1}{2} \end{array}\right]; \quad A^{-1} = \begin{bmatrix} 2 & 0 & -1 \\ \frac{1}{4} & \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

   (b) (1 point) Do the necessary multiplication to demonstrate that your answer is correct.

Check

$$A \cdot A^{-1} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 1 & 0 & 4 \end{bmatrix} \begin{bmatrix} 2 & 0 & -1 \\ \frac{1}{4} & \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 2-1 & 0 & -1+1 \\ \frac{1}{2}-\frac{1}{2} & 1 & -\frac{1}{2}+\frac{1}{2} \\ 2-2 & 0 & -1+2 \end{bmatrix} = I_3 \checkmark$$

   (c) (3 points) From you work in part (a), you should be able to **immediately** determine the LU-factorization of A. Write that factorization below.

$$L = E_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

check:

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}}_{U} = \underbrace{\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 1 & 0 & 4 \end{bmatrix}}_{A} \checkmark$$

2. (2 points) Explain how you know that the matrix $B = \begin{bmatrix} 1 & 2 & 3 \\ -5 & 4 & 2 \\ 2 & 4 & 6 \end{bmatrix}$ cannot be invertible.

- $\text{row}_3(B)$ is a multiple of $\text{row}_1(B)$.

OR

- elimination results in a row of all zeros.