 Name: Solutions

This quiz has six problems worth 10 points.

Below is the matrix A and its reduced row echelon form, R. Use these to answer the following questions.

$$A = \begin{bmatrix} 2 & -2 & 4 & 3 \\ 4 & -8 & 10 & 7 \\ 6 & 14 & 2 & 4 \end{bmatrix} \qquad R = \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \begin{bmatrix} 1 & 0 & 3/2 & 5/4 \\ 0 & 1 & -1/2 & -1/4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

1. (1 pt) What is the rank of A? 2

2. (1 pt) What are the *free variables* in this case? (Assume $\mathbf{x} = (x_1, x_2, x_3, x_4)$.) x_3, x_4

3. (2 pts) For each free variable, find a *special solution*.

for x_3: $x_3 = 1$, $x_4 = 0$

$x_1 + \frac{3}{2} = 0 \qquad x_1 = -3/2$

$x_2 - 1/2 = 0 \qquad x_2 = \frac{1}{2}$

$$\vec{s}_3 = \begin{bmatrix} -3/2 \\ 1/2 \\ 1 \\ 0 \end{bmatrix}$$

for x_4: $x_3 = 0$, $x_4 = 1$

$x_1 + 5/4 = 0 \qquad x_1 = -5/4$

$x_2 - 1/4 = 0 \qquad x_2 = 1/4$

$$\vec{s}_4 = \begin{bmatrix} -5/4 \\ 1/4 \\ 0 \\ 1 \end{bmatrix}$$

4. (2 pts) Efficiently and precisely describe $N(A)$, the null space of A.

$N(A) = \{ c\vec{s}_3 + d\vec{s}_4 : c, d \text{ real numbers} \}$

= all linear combinations of $\vec{s}_3$ and $\vec{s}_4$

$$A = \begin{bmatrix} 2 & -2 & 4 & 3 \\ 4 & -8 & 10 & 7 \\ 6 & 14 & 2 & 4 \end{bmatrix} \qquad R = \begin{bmatrix} 1 & 0 & 3/2 & 5/4 \\ 0 & 1 & -1/2 & -1/4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

5. (2 pts) Efficiently and precisely describe $C(A)$, the column space of A. (Note that A and R are copied above.)

$$C(A) = \left\{ c\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} + d\begin{bmatrix} -2 \\ -8 \\ 14 \end{bmatrix} : c, d \text{ real numbers} \right\}$$

$= \text{all linear combinations of } \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} \text{ and } \begin{bmatrix} -2 \\ -8 \\ 14 \end{bmatrix}$

6. (2 pts) Below is the augmented matrix $[A\ \mathbf{b}]$ for the system of equations $A\mathbf{x} = \mathbf{b}$ and its reduced row echelon form, R. Find the complete solution to the system $A\mathbf{x} = \mathbf{b}$.

$$A = \left[\begin{array}{cccc:c} 2 & -2 & 4 & 3 & 6 \\ 4 & -8 & 10 & 7 & 8 \\ 6 & 14 & 2 & 4 & 38 \end{array}\right] \qquad R = \left[\begin{array}{cccc:c} 1 & 0 & 3/2 & 5/4 & 4 \\ 0 & 1 & -1/2 & -1/4 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

x_p = a particular solution is $x_1 = 4,\ x_2 = 1,\ x_3 = x_4 = 0.$

$$\text{complete solution} = \left\{ \begin{bmatrix} 4 \\ 1 \\ 0 \\ 0 \end{bmatrix} + c\begin{bmatrix} -3/2 \\ 1/2 \\ 1 \\ 0 \end{bmatrix} + d\begin{bmatrix} -5/4 \\ 1/4 \\ 0 \\ 1 \end{bmatrix} : c, d \text{ real numbers} \right\}$$

$$= \left\{ \vec{x}_p + c\vec{S}_3 + d\vec{S}_4 : c, d \text{ real} \right\}$$