Linear Algebra Quiz 7 Name: Solutions

This quiz has six problems worth 10 points.

1. (4 points) Give examples of matrices for which the number of solutions to $A\mathbf{x} = \mathbf{b}$ is

   (a) 0 or 1, depending on $\mathbf{b}$. Explain your reasoning.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

A pivot in every column means that if it has a solution, that solution is unique. But a row of zeros means there may not be a solution (eg $\vec{b} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$)

   (b) 0 or ∞, depending on $\mathbf{b}$. Explain your reasoning.

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

The row of zeros means there may not be a solution. The free columm (col 3) means that there will always be an infinite number of solutions if solutions exist.

2. (4 points) Find a basis for each subspace below.

   (a) The subspace of 2×2 matrices consisting of all upper triangular matrices.

$$\begin{bmatrix} a & b \\ 0 & c \end{bmatrix} = a\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + b\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + c\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\text{basis} = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

   (b) The subspace of $\mathbb{R}^4$ consisting of all vectors orthogonal to $(1, 1, 0, 0)$ and $(1, 0, 1, 0)$.

want $(w\, x\, y\, z)$ so that

$$\begin{aligned} w + x \quad &= 0 \\ w \quad + y &= 0 \end{aligned}$$

If $w = 1$, then $x = y = -1$.

$$\text{basis} = \left\{ \begin{bmatrix} 1 \\ -1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

3. (2 points) [Fill in the blank.] Suppose A is an $m \times n$.

   (a) The vector $\mathbf{d}$ is in the row space of A when $A^T x = \vec{d}$ has a solution.

   (b) The row space of A is a subspace of $\mathbb{R}^n$.