This quiz has six problems worth 10 points.

1. (8 points) Let $A = \begin{bmatrix} 1 & 0 & 2 & -1 & 0 \\ 2 & 1 & 3 & 1 & 2 \\ 2 & 3 & 1 & 7 & 5 \\ 1 & -1 & 3 & -4 & -5 \end{bmatrix}$ and $\text{rref}(A) = R = \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array}$ $\begin{bmatrix} 1 & 0 & 2 & -1 & 0 \\ 0 & 1 & -1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

   (a) Find a basis for $C(A)$.

   $\underbrace{\begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix}}_{v_1}, \underbrace{\begin{bmatrix} 0 \\ 1 \\ 3 \\ -1 \end{bmatrix}}_{v_2}, \underbrace{\begin{bmatrix} 0 \\ 2 \\ 5 \\ -5 \end{bmatrix}}_{v_3}$

   (b) Find a basis for $N(A)$.

   x_3, x_4 free

   $(1, -3, 0, 1, 0)$

   $(-2, 1, 1, 0, 0)$

   (c) Find a basis for $C(A^T)$.

   $(1, 0, 2, -1, 0)$
   $(0, 1, -1, 3, 0)$
   $(0, 0, 0, 0, 1)$

   (d) Could the vector $\mathbf{v} = (-15, 10, -3, 1)$ be a basis for the left null space of A? Explain your reasoning.

   Yes. $\dim(N(A^T)) = \#\text{rows} - \dim(C(A)) = 4 - 3 = 1.$

   And $\vec{v} \perp C(A)$ since

   $\vec{v} \cdot \vec{v}_i = 0$

   $v \cdot v_1 = -15 + 20 - 6 + 1 = 0$

   $v \cdot v_2 = 0 + 10 - 9 - 1 = 0$ ✓

   $v \cdot v_3 = 0 + 20 - 15 - 5 = 0$

2. (2 points) [Fill in the blank.] The vectors $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \cdots \mathbf{a}_k$ form a basis for the vector space S if

1 the a_i's are linearly independent

and

2 the a_i's span S.