This quiz has 2 problems worth 10 points.

1. (3 points) Project the vector $\mathbf{b} = (1, -6, 8)$ onto the line through $\mathbf{a} = (2, 1, -3)$. Check that $\mathbf{e}$ is perpendicular to $\mathbf{a}$.

$$\vec{p} = \left(\frac{\vec{a}\cdot\vec{b}}{\vec{a}\cdot\vec{a}}\right)\vec{a} = \left(\frac{2-6-24}{4+1+9}\right)\begin{bmatrix}2\\1\\-3\end{bmatrix} = \frac{-28}{14}\begin{bmatrix}2\\1\\-3\end{bmatrix} = \begin{bmatrix}-4\\-2\\+6\end{bmatrix}$$

$$\vec{e} = \vec{b} - \vec{p} = \begin{bmatrix}1\\-6\\8\end{bmatrix} - \begin{bmatrix}-4\\-2\\6\end{bmatrix} = \begin{bmatrix}5\\-4\\2\end{bmatrix}$$

check $\vec{e}\cdot\vec{a} = 10 - 4 - 6 = 0$ ✓ $\vec{e} \perp \vec{a}$

2. Let $(1,0),\ (2,1),\ (3,4),\ (5,1)$ be four points in the xy-plane.

(a) (3 points) Write down the unsolvable equations for a line $y = C + Dx$ through the four points.

$C + D = 0$

$C + 2D = 1$

$C + 3D = 4$

$C + 5D = 1$

$1+4+9+25 = 14+25 = 39$

(b) (3 points) Set up the normal equations to solve for $\hat{x}$.

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 5 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 0 \\ 1 \\ 4 \\ 1 \end{bmatrix} \quad A^T A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 5 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 4 & 11 \\ 11 & 39 \end{bmatrix}$$

$$A^T \vec{b} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 5 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 2+12+5 \end{bmatrix} = \begin{bmatrix} 6 \\ 19 \end{bmatrix}$$

Normal equ: $\begin{bmatrix} 4 & 11 \\ 11 & 39 \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} 6 \\ 19 \end{bmatrix}$

(c) (1 points) In fact, $\hat{x} = (0.7143, 0.2857)$. What does this tell you?

The best fit line for these points is

$y = 0.7143 + 0.2857x$