

The exam will be Wednesday October 1, 9:15-10:15. No books, notes, calculator, electronics, or internet access.

It will cover Chapters 1 and 2.

### Chapter 1: Introduction to Vectors

#### Section 1.1: Vectors and Linear Combinations

terminology/definitions: vector, linear combinations of vectors, vector addition, scalar multiplication, a vector equation.

skills: how to draw vector addition, subtraction, scalar multiplication, and linear combinations

#### Section 1.2: Lengths and Dot Products

terminology/definitions: the dot product of two vectors, the length of a vector, a unit vector, the angle between two vectors

skills: how to use the dot product to determine the angle between two vectors (or whether they are orthogonal, acute, obtuse); how to find a unit vector in the same direction as a given vector, how to find the length of a vector.

#### Section 1.3: Matrices

terminology/definitions: matrices, matrix - vector product (both a row-view and a column-view), matrix-vector equations (both a row-view and a column-view), a first look at the idea of a matrix inverse, a first look at the idea of a set of vectors being linearly independent or dependent.

skills: how to determine if one vector is a linear combination of others, how to solve systems of equations (in an ad hoc manner, perhaps), to recognize when a system of equations has one solution or multiple solutions or no solution.

### Chapter 2: Solving Linear Equations

#### Section 2.1: Vectors and Linear Equations

terminology/definitions: The column and row pictures of  $Ax = b$  both algebraically and geometrically, in 2- and 3-dimensions, the identity matrix, a first look at matrix multiplication.

skills: Understand the geometric and numerical consequences when solving  $Ax = b$  and  $b = 0$ , understand the geometric and numerical interpretations of solutions to  $Ax = b$

#### Section 2.2: The Idea of Elimination

terminology/definitions: elimination, multiplier (ex:  $\ell_{31}$ ), elimination matrix, pivot,

skills: Know how to use elimination, including elimination matrices, to transform the matrix  $A$  or the system of equations  $Ax = b$  into upper triangular form, recognize when the process of elimination breaks down.

#### Section 2.3: Elimination Using Matrices

terminology/definitions: the formal (rigid) algorithm for elimination, augmented matrix, row exchanges and corresponding matrices, formal matrix multiplication and its algebra

skills: Perform elimination in this formal sense. Be able to find the corresponding elimination matrices and the single matrix that performs all steps simultaneously.

#### Section 2.4: Rules for Matrix Operation

terminology/definitions/rules: there are several rules about how matrix multiplication does and does not behave (ex: it's associative and it distributes through addition, but it's not necessarily commutative), block multiplication

skills: This was a deep dive into matrix multiplication. You should be able to multiply matrices using any of the four approaches.

#### Section 2.5: Inverse Matrices

terminology/definitions/rules: the inverse of a matrix, algebra of inverses (ex  $(AB)^{-1} = ??$ )

skills: Gauss-Jordan Elimination to find the inverse of a matrix  $A$ . How the inverse relates to solutions to systems of equations.

#### Section 2.6: Elimination = Factorization: $A = LU$

terminology/definitions:  $LU$ -factorization

skills: How to find an  $LU$ -factorization for a matrix  $A$  or recognize that no such factorization exists, How this factorization relates to elimination. How the  $L$  and  $U$  matrices can be used to solve  $A\mathbf{x} = \mathbf{b}$ .

#### Section 2.7: Transposes and Permutations

terminology/definitions: the transpose of a matrix, transpose algebra, symmetric matrices, permutation matrices