The exam will be Wednesday November 12, 9:15-10:15. No books, notes, calculator, electronics, or internet access.

It will cover Chapters 3 and 4 and Section 5.1 on Determinants.

Chapter 3: Vector Spaces and Subspaces
Section 3.1: Spaces of Vectors

terminology/definitions:
a *vector space* and a *subspace* of a vector space,
$\mathbb{R}^n$ as a vector space
the vector spaces $\mathbf{M}, \mathbf{F}, \mathbf{Z}$ (real 2 by 2 matrices, real functions $f(x)$ and the zero vector),
defining a subspace as the set of all linear combinations of a set of vectors
column space of a matrix, $C(A)$
a subspace *spanned* by a set of vectors,

skills/principles:
identify sets that are or are not subspaces of standard vector spaces,
the geometric properties of subspaces (when compared to sets that are not subspaces),
the equivalence of the statements $Ax = b$ is solvable and b is in the column space of A.

Section 3.2: The Nullspace of A: Solving $Ax = 0$ and $Rx = 0$

terminology/definitions:
the *null space of a matrix* A, $N(A)$
reduced row echelon form of a matrix, typically R (and this is different from U, notation for the upper triangular form)
pivot and *free* columns
special solution associated with a free column
rank of a matrix and what it indicates about a matrix, solutions to $Ax = b$, and free columns

skills/principles:
know how to find a reduced row echelon form of a matrix
know how to efficiently find the null space of a matrix
know that row operations do not change the null space

Section 3.3: The Complete Solutions to $Ax = b$

terminology/definitions:
a *complete* solution to $Ax = b$ and how to find it efficiently,
full column rank or *full row rank* and what these indicate about $N(A)$ or $C(A)$
skills/principles:
At the end of this section, we are supposed to understand in great detail what the reduced row echelon form of A should indicate about the nature of solutions to $Ax = b$, $C(A)$, and $N(A)$.

Section 3.4: Independence, Basis, and Dimension

terminology/definitions:
linearly independent sets of vectors and *linearly dependent* sets of vectors
a spanning set of vectors

basis
row space of matrix A, $C(A^T)$
dimension of a vector space

skills/principles:
how to show (rigorously) that at set of vectors is linearly independent or linearly dependent
how to show (rigorously) that a set of vectors spans a vector space (or subspace)
how to find bases for subspaces, like $C(A)$ or $C(A^T)$ or $N(A)$
how to determine the dimension of subspaces

Section 3.5: Dimensions of the Four Subspaces

terminology/definitions:
the *four subspaces* in words: *column space*, *row space*, *null space* and *left null space* and in symbols: $C(A), C(A^T), N(A), N$

skills/principles:
know how to find each of the four subspaces, find bases for each of them, determine their dimension, and understand their geometry (what space to they live in? orthogonality?)
how elimination does (or does not) change the four subspaces
know the basic principles on page 184 of your text. (You don't need to be able to state the text's "Fundamental Theorem of Linear Algebra", but you should definitely know it!)

Chapter 4: Orthogonality
Section 4.1: Orthogonality of the Four Subspaces

terminology/definitions:
orthogonal vectors and the algebraic consequences
orthogonal vector spaces, and examples
orthogonal complements

skills/principles:
If one knows a vector space, V, has dimension d and one has a set S of d vectors in V, one can know that S is a basis of V by checking *one* of the two properties of a basis (ie check S is linearly independent OR check S spans V.) One does not have to check both.

Section 4.2: Projections

terminology/definitions:
the *projection*, $\mathbf{p}$, of a vector $\mathbf{b}$ onto a vector $\mathbf{a}$
the *projection*, $\mathbf{p}$, of a vector $\mathbf{b}$ onto a subspace
the *error* $\mathbf{e} = \mathbf{b} - \mathbf{p}$
$\mathbf{P}$, the projection matrix
$\widehat{x}$, the coefficient vector used to construct the projection $\mathbf{p}$
skills/principles:
know how to compute the projection of one vector onto another and how to draw the projection, and the same for the error
know how to compute the projection of one vector onto a subspace and how to draw the projection, and the same for the error
know that the projection minimizes the error as measured by the sum of the squares of the differences

in components
know how to compute $\mathbf{P}$, its properties, and how to use it
geometrically, we think of $\mathbf{p}$ as the vector in S closest to the vector $\mathbf{b}$

Section 4.3: Least Squares Approximations

terminology/definitions:
the *normal equations*: $\mathbf{A}^T\mathbf{A}\mathbf{x} = \mathbf{A}^T\mathbf{b}$, whose solution is $\widehat{\mathbf{x}}$

skills/principles:
the projection $\mathbf{p}$ (from 4.2) and the methods to find it can be reinterpreted to finding best solutions to over-determined systems of equations or to finding curves of best fit to a given data set.
when $\mathbf{A}\mathbf{x} = \mathbf{b}$ has no solution, $\widehat{\mathbf{x}}$, is the "best" solution, or the "least squares" solution because it minimizes the error $|\mathbf{A}\mathbf{x} - \mathbf{b}|^2$
the least squares solutions can be used to find the line (or parabola or cubic...) of best fit to a set of points (data)

Section 4.4: Orthonormal Bases and Gram-Schmidt

terminology/definitions:
orthogonal basis
orthonormal basis
Gram-Schmidt Process
skills/principles:
how to find the projection of a vector $\mathbf{b}$ onto a subspace S given an orthonormal basis $\mathbf{q}_1, \cdots \mathbf{q}_n$
how to use the Gram-Schmidt Process to find an orthogonal or orthonormal basis from a given basis
If $\mathbf{Q}$ has orthonormal columns, then know the many efficiencies that result. (ex: $\mathbf{Q}^T\mathbf{Q}$ =?, finding $\widehat{\mathbf{x}}$, finding $\mathbf{P}$)

Chapter 5 Section 1: The Properties of Determinants

terminology/definitions:
determinant of a 2 by 2 matrix
skills/principles:
There are 10 principles to know about the determinant (listed in the text)
Know how to apply these to an n by n matri