(1) Suppose $f : \mathbf{R}^3 \to \mathbf{R}^2$ is a linear transformation and

$$f(1,0,0) = (1,-3),\ f(0,1,0) = (-2,6),\ f(0,0,1) = (0,0).$$

(a) Find $f(3,4,5)$.

(b) Find a matrix A such that $f(\mathbf{x}) = A\mathbf{x}$, for all $\mathbf{x}$ in $\mathbf{R}^3$.

(c) Find the kernel and range of f

(2) Suppose the function $g : \mathbf{R}^2 \to \mathbf{R}^2$ is defined as $g(x_1, x_2) = (2x_1 + 3x_2, x_1 + 4x_2)$.

(a) Compute $g(3,1)$.

(b) Show that g is a linear transformation by finding a matrix A such that $f(\mathbf{x}) = A\mathbf{x}$, for all $\mathbf{x}$ in $\mathbf{R}^2$.

(c) Compute $g(3,-1)$ and $g(1,1)$. What does this tell you about matrix A? (List as many answers to this question as you can think up!)

(d) Find a (different) matrix, say B, such that $g(\mathbf{x}) = B\mathbf{x}$ *under the assumption that all vectors in* $\mathbf{R}^2$ *are written in terms of the basis:* $\mathbf{v}_1 = (3,-1)$, $\mathbf{v}_2 = (1,1)$.

(e) Use the matrix B to find the image of $\mathbf{w} = (3,1)$ under g and confirm it is the same as your computation in part (a).