(1) Find a basis for the four subspaces of A.

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 & -2 \\ 1 & 2 & -2 & 0 & 0 \\ 1 & 2 & -1 & 1 & -4 \\ 2 & 4 & 3 & -3 & 2 \end{bmatrix} \quad \to \quad R = \begin{bmatrix} 1 & 2 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

$$A^T \quad \to \quad R = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(2) Why are the principles below true?

(a) If $\mathbf{v}$ and $\mathbf{w}$ are orthogonal, then $\|\mathbf{v}+\mathbf{w}\|^2 = \|\mathbf{v}\|^2 + \|\mathbf{w}\|^2$.

(b) If $\mathbf{v}$ and $\mathbf{w}$ are orthogonal, then $\mathbf{v}$ and $\mathbf{w}$ are linearly independent.

(c) For any matrix A, $C(A^T) \perp N(A)$.

(d) For any matrix A, $C(A) \perp N(A^T)$.

(e) If vector space V has two bases $B_1 = \{v_1, v_2, \cdots, v_m\}$ and $B_2 = \{w_1, w_2, \cdots, w_n\}$, then $m = n$. (That is, the notion of the dimension of a vector space is well-defined.)

(f) If the dimension of a vector space, V, is d, then you can tell that a set of d vectors is a basis of V by checking whether it is...