

- (1) Find the eigenvalues $\lambda_1 \leq \lambda_2 \leq \lambda_3$ and associated eigenvectors, $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3$ for the matrix \mathbf{A} below.

$$\mathbf{A} = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

(2) Define the two matrices $\mathbf{X} = [\mathbf{x}_1 \ \mathbf{x}_2 \ \mathbf{x}_3]$ and $\mathbf{\Lambda}$ to be a 3 by 3 *diagonal* matrix with $\lambda_1, \lambda_2, \lambda_3$ down the main diagonal *in that order* and write them below.

(a) Is \mathbf{X} invertible or not? Why?

(b) Compute $\mathbf{A}\mathbf{X}$.

(c) Compute $\mathbf{X}\mathbf{\Lambda}$.

(d) Explain, without computation, that $\mathbf{A} = \mathbf{X}\mathbf{\Lambda}\mathbf{X}^{-1}$

(e) Use your answer from part (d), to find another way to compute \mathbf{A}^5 . Are there advantages to the alternative approach?