1. Text 12.2 Modified. Suppose that the $m \times n$ matrix Q has orthonormal columns, say $q_1, q_2, \cdots, q_n$.

a) Why do you immediately know that $m \geq n$ and hence Q is tall or square?

b) Suppose that b is an m-vector. Show that $\hat{x} = Q^T b$ is the vector that minimizes $J(x) = \|Qx - b\|^2$. **Do this by setting up the normal equations directly.**

c) Rewrite your solution for $\hat{x}$ in part b **in terms of the columns of** Q. It will be helpful to write $b = (b_1, b_2, \cdots, b_m)$. You should now be able to complete the sentence below using the words "inner product...".

> If the columns of Q are orthonormal, then the least squares approximation $\hat{x}$ for the system $Qx = b$ is ...

2. Supplemental **12.6** Hint: Your matrix A will be 6×4 and will have a lot of 0's and 1's.

3. Suppose we want to compute the coefficients of the quadratic $p(t) = c_1 + c_2 t + c_3 t^2$ that is the least squares fit to the following (t, y) data points: $(0, 0)$, $(1, 8)$, $(3, 8)$, $(4, 20)$.

1. Formulate the problem in the form of $Ac = b$. That is, find the matrix A and the vector b. You don't need to solve this system.
2. Now formulate the normal equation $A^T Ac = A^T b$ that you would use to find these best-fit coefficients. Again, do not solve the system. (But don't worry, there is a Julia problem that tackles this!)

4. Text: problem 12.4. Hint: you will find that the square roots of the weights are important and note that the weights are *positive*. For part (b), remember that the columns of C are linearly independent if and only if the only solution of $Cx = 0$ is $x = 0$.