1. Below is a system of first-order, linear, differential equations. Rewrite this system in the form $Au = \frac{du}{dt}$, $u = u(0)$ for $t = 0$. (That is, you need to define u, A, and $u(0)$. You are *not* being asked to solve the system.)

$$\frac{dv}{dt} = 4v - 5w \qquad v = 8 \text{ when } t = 0$$

$$\frac{dw}{dt} = 2v - 3w \qquad w = 5 \text{ when } t = 0$$

2. Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 1 & 2 & 3 & 6 & 9 \\ 2 & 4 & 5 & 10 & 16 \end{bmatrix}$$

Find the null space of A by using elementary row operations to produce the reduced row echelon form of A. (You will need to record these for problem 4 later. If you are efficient, you will use a total of 5 row operations.)

3. For the same matrix A as in the previous problem, one solution of $Ax = (1, 3, 6)$ is $x = (5, -4, 3, -2, 1)$. Find all solutions to $Ax = (1, 3, 6)$.

4. We now return to the matrix A of problem 2.

a) Demonstrate that if $E_1 = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then $E_1 A$ is the result of doing an elementary row operation on A that involves leaving rows 1 and 3 unchanged, but replacing the second row of A with $row_2(A) - row_1(A)$. Indeed E_1 stands for "first elementary row operation."

b) Find E^{-1} (which is easy!).

c) Find the matrices E_1, E_2, E_3, E_4, and E_5 for each of **your** row operations from problem 2.

d) Use a computational tool to find the product $B = E_5 E_4 E_3 E_2 E_1$ and verify that BA is A in reduced row echelon form.

e) Explain why you know that B is invertible.

f) Let $C = rref(A)$, the reduced row echelon form of A. Write A in terms of B and C.

g) Part (f) above is another way of factoring a matrix A (called LU-factorization) and it can also be used to solve equations. You don't have to do anything for this part. I put it in so you would know why you were asked to do this.

5. Suppose A is an $m \times n$ matrix and that W is an invertible $m \times m$ matrix.

a) Show that the null space of A and the null space of WA are the same as each other. (One strategy is to pick a vector in $N(A)$ and show it must be in $N(WA)$. Then reverse that process.)

b) What can you conclude about the null space of WA if you don't know that W is invertible?