1. A 4×4 matrix has $\det(A) = 1/3$. Find $\det(2A)$, $\det(-A)$, $\det(A^2)$ and $\det(A^{-1})$.

2. Find two 2×2 matrices A and B with $\det(A) = 1$ and $\det(B) = 1$ but $\det(A + B) = 0$. So there is no rule $\det(A + B) = \det(A) + \det(B)$.

3. Compute the determinant of

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 6 & 6 & 1 \\ -1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 7 \end{bmatrix}$$

by reducing to an upper triangular matrix.

4. Compute the determinant of

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 10 \\ 5 & 6 & 7 \end{bmatrix}$$

by reducing to an upper triangular matrix. Note that you will need to keep track of row interchanges.

5. Find the determinant of

$$C = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 4 & 5 & 6 \end{bmatrix}.$$

Then explain your result.

6. Compute the determinant of

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 4 & 4 \\ 5 & 6 & 7 \end{bmatrix}$$

by expansion along the third row, i.e. the long formula with six terms coming from three 2×2 determinants.

7. Compute the determinant of

$$A = \begin{bmatrix} 5 & 1 & -1 & 2 & 1 \\ 3 & 0 & 0 & 0 & 3 \\ 2 & 3 & 0 & 1 & 2 \\ 0 & 0 & 1 & 0 & 1 \\ 6 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

You'll want to choose your expansion row wisely...