1. For each matrix below, (i) determine its eigenvalues, (ii) determine $N(A - \lambda I_n)$ for every **real** eigenvalue λ, and (iii) pick a particular eigenvector associated with every real eigenvalue.

a) $A = \begin{bmatrix} 3 & -5 \\ 1 & -3 \end{bmatrix}$

b) $B = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

c) $E = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

d) $C = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix}$

e) D an $n \times n$ diagonal matrix

2. For matrix A in problem 1 part a, confirm that **trace**(A) is the sum of the eigenvalues and $\det(A)$ is the product of the eigenvalues.

3. Suppose F is an upper triangular matrix. Can you easily determine its eigenvalues? Its associated eigenvectors?

4. This problem refers to matrix C in problem 1 part (d). Note that you should have obtained 3 distinct eigenvalues and picked three distinct associated eigenvectors in part (iii).

a) Explain why you know (without doing any additional computation) that the three eigenvectors you obtained are linearly independent.

b) Let P be the 3×3 matrix with the eigenvectors as columns. Explain how you know that P is invertible.

c) Use a computational tool to find P^{-1}.

d) Use a computational tool to compute $P^{-1}AP$. What do you observe?

5. This problem revisits problem 1 from Homework 11, which is restated below.

Below is a system of first-order, linear, differential equations. Rewrite this system in the form $Au = \frac{du}{dt}$, $u = u(0)$ for $t = 0$. (That is, you need to define u, A, and $u(0)$. You are *not* being asked to solve the system.)

$\frac{dv}{dt} = 4v - 5w \qquad v = 8$ when $t = 0$

$\frac{dw}{dt} = 2v - 3w \qquad w = 5$ when $t = 0$

a) Find the eigenvalues of the matrix A.

b) Find an associated eigenvector for each eigenvalue from part (a).

c) Find the two pure exponential solutions to the differential equation $Au = \frac{du}{dt}$ and verify that they are solutions.

d) Solve the system $Au = \frac{du}{dt},\ u = u(0)$ for $t = 0$.

e) Determine $u(5)$.

f) Describe the long-term behavior of $u(t)$.