**1.** Let

$$x_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \quad x_2 = \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix}, \quad x_3 = \begin{bmatrix} -1 \\ 5 \\ 4 \end{bmatrix}.$$

Show that $x_1$, $x_2$ and $x_3$ are linearly dependent two different ways:

a) Find coefficients $\beta_1, \beta_2, \beta_3$ such that $\beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 = 0$.
(There are many ways to do this. You can try just guessing or a more systematic approach.)

b) Write $x_1$ as a linear combination of $x_2$ and $x_3$.

**2.** Let

$$y_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad y_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad y_3 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}.$$

a) Show that $y_1$, $y_2$ and $y_3$ are linearly independent. That is, show that if $\beta_1$, $\beta_2$ and $\beta_3$ are numbers such that $\beta_1 y_1 + \beta_2 y_2 + \beta_3 y_3 = 0$ then, in fact, $\beta_1 = \beta_2 = \beta_3 = 0$.
(Note that you *must* be systematic about this.)

b) Briefly explain why $y_1$, $y_2$ and $y_3$ form a basis for $\mathbb{R}^3$. Your answer should be one sentence.

c) Because these vectors form a basis for $\mathbb{R}^3$, and because $z = (2, 1, 3)$ is a vector in $\mathbb{R}^3$, there is a unique linear combination $\beta_1 y_1 + \beta_2 y_2 + \beta_3 y_3 = z$. Find the numbers $\beta_1, \beta_2$ and $\beta_3$.
(This will be tedious. We will automate this later. This one time, try doing it by hand. Practice efficiency!)

**3.** In class I mentioned "Fact A". Your book calls this the "independence-dimension inequality". It says "A linearly independent collection of $n$ vectors has at most $n$ elements".

a) Consider

$$a_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad a_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad a_3 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$$

Explain how you know, without doing any work, that this collection is linearly dependent.

b) Because the collection is linearly dependent, it has redundancy. Exhibit this redundancy by finding *three* different linear combinations of the vectors that give you $(0, 0)$.
One of these is trivial! One will take a little bit of work. Once you have that one, you can easily find infinitely many others, so locating a third will be a breeze!

c) Exhibit the redundancy differently by finding *three* different linear combinations of $a_1$, $a_2$ and $a_3$ that give you $(4, 7)$.
**Hint:** Find one linear combination that works. Then use you answer from part **(a)** to help!

**4.** Suppose $w_1$, $w_2$ and $w_3$ are any vectors at all in $\mathbb{R}^{17}$. Let $v_1 = w_1 - w_2$, $v_2 = w_2 - w_3$ and $v_3 = w_3 - w_1$. Show that $v_1$, $v_2$ and $v_3$ are linearly dependent. **Hint:** find an explicit linear combination that yields zero.

**5.** Text: 5-4

**6.** Text: 5-5 modified as follows. Suppose $a$ and $b$ are any $n$-vectors. Find a formula in terms of $a$ and $b$ for a scalar $\gamma$ such that $a-\gamma b$ is perpendicular to $b$. Then draw a picture of $a$, $b$, and $a - \gamma b$ when $a = (0, 1)$ and $b = (1, 1)$.