

1. Let

$$a = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \quad c = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix},$$

- a) Perform the Gram-Schmidt algorithm on these vectors (in this order) to determine orthonormal vectors q_1, q_2 and q_3 .
- b) Write the vector $d = (1, 1, 1, -3)$ as a linear combination of q_1, q_2 and q_3 . Recall that because the q_i 's are orthonormal, the coefficients of the linear combination are given by $q_i^T d$.
- c) From part (b) you know that d is also a linear combination of the vectors a, b and c . In fact, this is easy to spot. Using whatever technique you would like, write d as such a linear combination.
- d) If we performed Gram-Schmidt on the collection of vectors a, b, c and d , what would have happened? Be specific in your answer.