This quiz is worth 10 points.

1. (4 points) Determine whether each of the following scalar-valued functions of n-vectors is linear. If it is a linear function, give its inner product representation (i.e. an n vector a for which $f(x) = a^T x$ for all x). If it is not linear, give specific x, y, α, and β for which superposition fails:

$$(i.e.\ f(\alpha x + \beta y) \neq \alpha f(x) + \beta f(y)).$$

   (a) $f(x)$ is the average of the first 3 entries of vector x. You an assume $n \geq 3$.

It is linear.

$a = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \text{zeros}(n-3))$

   (b) $f(x)$ is minimum entry of x. That is $f(x) = \min\{x_1, x_2, x_3, \cdots x_n\}$.

It is not linear. Pick $x = (1,-1)$, $y = (-1,1)$, $\alpha = \beta = 1$.

$f(\alpha x + \beta y) = f(0,0) = 0$

$\alpha f(x) + \beta f(y) = 1(-1) + 1(-1) = -2$

These are not equal.

2. (2 points) Suppose $f : \mathbb{R}^3 \to \mathbb{R}$ is a **linear function**. Further, suppose, $f(2,-4,3) = 10$ and $f(2,1,0) = 8$. Determine the value of $f(2,-9,6)$ if possible. If this is not possible, explain why.

We need to write $(2,-9,6) = \alpha(2,-4,3) + \beta(2,1,0)$.

So $\alpha = 2$. If $\alpha = 2$, $\beta = -1$. Quick check: $2(2,-4,3) - (2,1,0) = (2,-9,6)$

So $f(2,-9,6) = 2 \cdot f(2,-4,3) - 1 \cdot f(2,1,0)$

$= 2(10) - (8) = 12$