This quiz is worth 10 points.

1. (2 points) Suppose a and b are n-vectors. Show that if $a \perp b$, then $\|a+b\| = \sqrt{\|a\|^2 + \|b\|^2}$.

$\|a+b\| = \sqrt{(a+b)^T(a+b)} = \sqrt{a^Ta + 2a^Tb + b^Tb}$

$= \sqrt{\|a\|^2 + \|b\|^2}$ b/c $a^Tb = 0$

2. (8 points) Suppose $a = (1,1,0)$ and $b = (0,1,1)$.

(a) Is the angle between a and b acute, obtuse or a right angle?

$a^Tb = 0+1+0 > 0$. So θ is acute

* More detailed explanation →

(b) Write an equation for the line L between a and b. (Recall that the line determined by two points a and b is given by $L(t) = (1-t)a + tb$.)

$L(t) = (1-t)\begin{bmatrix}1\\1\\0\end{bmatrix} + t\begin{bmatrix}0\\1\\1\end{bmatrix} = \begin{bmatrix}1-t\\1\\t\end{bmatrix}$

(c) Show that the point $P = (3,1,-2)$ lies on the line L determined by a and b.

For $t=-2$, $L(-2) = \begin{bmatrix}1+2\\1\\-2\end{bmatrix} = \begin{bmatrix}3\\1\\-2\end{bmatrix} = P$

* more detailed explanation →

(d) Let X be the point $(4,0,6)$. Show that P is the point on L that is closest to the point X.

Typo!
$Y = (7,1,4)$
(ie: $Y = X+P$)

Answer to the correct problem:
Check that $(Y-P) \perp P$.
So $Y-P = (4,0,6)$.

Now $(Y-P)^TP = \begin{bmatrix}4\\0\\6\end{bmatrix}^T\begin{bmatrix}3\\1\\-2\end{bmatrix} = 12-12=0$

(e) Determine the distance between X and P.

$\|X-P\| = \|(4-3, 0-1, 6-(-2)\| = \|(1,-1,8)\| = \sqrt{1+1+64} = \sqrt{66}$

2a. In class & in text we observed/read

- If $a^T b > 0$, θ is acute
- If $a^T b < 0$, θ is obtuse
- If $a^T b = 0$, θ is right.

These simple observations do follow from

$$\cos(\theta) = \frac{a^T b}{\|a\| \|b\|}$$

2c. This question asks us to confirm the existence of some t so that $L(t) = (3,1,-2)$.
That is, find t so that

$$(1-t)\begin{bmatrix}1\\1\\0\end{bmatrix} + t\begin{bmatrix}0\\1\\1\end{bmatrix} = \begin{bmatrix}3\\1\\-2\end{bmatrix}$$

Looking only at the last coordinate, we see immediately that $t = -2$.