This quiz is worth 10 points. Name: Solutions

1. (2 points) Very briefly, explain how you can conclude *without any computation* that the vectors below are linearly **dependent**.

$$v_1 = \begin{bmatrix}1\\2\\3\\4\end{bmatrix}, v_2 = \begin{bmatrix}2\\1\\4\\3\end{bmatrix}, v_3 = \begin{bmatrix}1\\-1\\1\\0\end{bmatrix}, v_4 = \begin{bmatrix}1\\0\\1\\0\end{bmatrix}, v_5 = \begin{bmatrix}\pi\\\sqrt{2}\\0\\1\end{bmatrix}$$

The number of vectors (5) is larger than the number of entries in the vector (4). This violates Fact A: The number of linearly independent n-vectors is at most n.

2. (5 points) Show the vectors $a_1 = \begin{bmatrix}1\\0\\0\\1\end{bmatrix}, a_2 = \begin{bmatrix}1\\1\\1\\0\end{bmatrix}, a_3 = \begin{bmatrix}0\\-1\\1\\1\end{bmatrix}$ are linearly **independent.**

This explains what $\beta_1, \beta_2, \beta_3$ are.

Suppose

$\beta_1 a_1 + \beta_2 a_2 + \beta_3 a_3 = 0$

Then

eq1 $\beta_1 + \beta_2 = 0$

eq2 $\beta_2 - \beta_3 = 0$

eq3 $\beta_2 + \beta_3 = 0$

eq4 $\beta_1 + \beta_3 = 0$

This shows how equations 1-4 imply β_1, β_2 and β_3 must <u>all</u> be zero

adding equations 2 and 3 we obtain:

$2\beta_2 = 0$.

So $\beta_2 = 0$.

Since $\beta_2 = 0$, eq1 implies $\beta_1 = 0$

Since $\beta_2 = 0$, eq2 implies $\beta_3 = 0$.

Thus, the only way a linear combination of a_1, a_2, and a_3 can equal is if all coefficients, β_1, β_2, and β_3, are zero.

→ This translates the linear combination of a_i's into a system of equations in β_i's

3. (3 points) Show that the vectors $x_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, x_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, x_3 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ are linearly **dependent**.

   (Note: You don't need to look too hard.)

$$\boxed{x_1 - x_2 - x_3 = 0}$$

← This was obtained by inspection (ie. looking)

Since the coefficient in front of x_1 is $1 \neq 0$, we have shown the set x_1, x_2, x_3 is linearly dependent.

Give a particular linear combination of the vectors

Explain/identify what about the linear combination is important.

4. (1 point bonus) Answer Question 3 above in a different way.

(Many options here.)

- Observe that for $k \neq 0$,

  $$kx_1 - kx_2 - kx_3 = 0$$

  So there are an infinite number of linear combinations of the x_is that sum to the zero vector.

- Observe $x_1 = x_2 + x_3$.

  So one vector (x_1) can be written as a linear combination of the others.