This quiz is worth 10 points.   Name: _____

1. (3 points) Perform elementary row operations on the matrix below to obtain a matrix in reduced row echelon form. For full points, you need to state explicitly what row operations you are performing. Note that you don't need more than three steps.

— zero here

$$\begin{bmatrix} 1 & 0 & 4 & 16 \\ 0 & 1 & 2 & 2 \\ 0 & 2 & 5 & 7 \end{bmatrix} \xrightarrow{r_3 = r_3 - 2r_2} \begin{bmatrix} 1 & 0 & 4 & 16 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \xrightarrow{r_2 = r_2 - 2r_3} \begin{bmatrix} 1 & 0 & 4 & 16 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

↑ zero here (under the 2 in row 3)

```
  0 -2 -4 -4
  0  2  5  7
 _____
  0  0  1  3
```

zero here

```
  0  1  2  2
  0  0 -2 -6
 _____
  0  1  0 -4
```

$$\xrightarrow{r_1 = r_1 - 4r_3} \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 3 \end{bmatrix} \checkmark$$

```
 1  0  4   16
 0  0 -4  -12
 _____
 1  0  0    4
```

2. (3 points) The system of equations $S = \begin{cases} w - 2x - 6y - 2z = -1 \\ x + 3y + 2z = 0 \\ 2w - x - 3y + 2z = -2 \end{cases}$ has augmented matrix

$A = \begin{bmatrix} 1 & -2 & -6 & -2 & -1 \\ 0 & 1 & 3 & 2 & 0 \\ 2 & -1 & -3 & 2 & -1 \end{bmatrix}$. Use the reduced row echelon form of the matrix A, given below, to solve the system of equations S.

$$w \quad x \quad y \quad z \quad \text{← constant}$$

$rref(A) = \begin{bmatrix} 1 & 0 & 0 & 2 & -1 \\ 0 & 1 & 3 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$w + z = -1$
$x + 3y + 2z = 0$

So y and z can be any real number.

ANS: $\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 - z \\ -3y - 2z \\ y \\ z \end{bmatrix}$ for any values y and z.

3. (4 points) Let $a = (1, 0, 1)$ and $x = (x_1, x_2, x_3, x_4, x_5)$. Find $a * x$, the convolution of a and b.

Since a has dimension 3 and x has dimension 5, we know $a * x$ will have dimension $3 + 5 - 1 = 7$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_1 + x_3 \\ x_2 + x_4 \\ x_3 + x_5 \\ x_4 \\ x_5 \end{bmatrix}$$

7×5