This quiz is worth 10 points.

Name: Solutions

1. (4 points) The 4-vector $c = (c_1, c_2, c_3, c_4)$ represents the coefficients of a cubic polynomial $p(x) = c_1 + c_2x + c_3x^2 + c_4x^3$. Express the conditions

$$\underset{①}{p(0) = 1},\ \underset{②}{p'(0) = 0},\ \underset{③}{p(2) = 1},\ \underset{④}{p'(2) = 0}$$

as a set of linear equations of the form $Ac = b$.

$$p(x) = c_1 + c_2 x + c_3 x^2 + c_4 x^3$$

$$p'(x) = c_2 + 2c_3 x + 3c_4 x^2$$

① $p(0) = c_1 = 1$

② $p'(0) = c_2 = 0$

③ $p(2) = c_1 + 2c_2 + 4c_3 + 8c_4 = 1$

④ $p'(2) = c_2 + 4c_3 + 12c_4 = 0$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 2 & 4 & 8 \\ 0 & 1 & 4 & 12 \end{bmatrix},\quad c = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix},\quad b = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Bonus: Use your knowledge of Calculus to solve the system of equations above or determine that no solution is possible.

No polynomial of degree 3 or less can have two critical numbers with the same y-value unless it's a horizontal line. So the only solution is $c_1 = 1$, $c_2 = c_3 = c_4 = 0$.

2. (6 points) Which of the following are linear or affine functions $f : \mathbb{R}^{\nvDash} \to \mathbb{R}^3$? For ones which are linear, express them in the form $f(x) = Ax$ for some specific matrix A. For ones which are affine but not linear, express them in the form $f(x) = Ax + b$ for some specific matrix A and vector b. For ones which are not affine, demonstrate your conclusion is correct but selecting an appropriate example.

(a) $f(x_1, x_2) = (x_2, x_2 - x_1, x_2 x_1)$

not affine

example: $u = (1,1)$ $v = (0,0)$ $\alpha = \beta = \frac{1}{2}$

$\alpha u + \beta v = (\frac{1}{2}, \frac{1}{2})$

not equal

$f(\frac{1}{2}, \frac{1}{2}) = (\frac{1}{2}, 0, \frac{1}{4})$

$\alpha f(u) + \beta f(v) = \frac{1}{2} f(1,1) = \frac{1}{2}(1,0,1) = (\frac{1}{2}, 0, \frac{1}{2})$

(b) $f(x_1, x_2) = (0, x_1, \frac{x_1+x_2}{3})$

linear

$$A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

(c) $f(x_1, x_2) = (\frac{x_1+1}{2}, \frac{x_2+1}{2}, x_1)$ $= \begin{bmatrix} \frac{x_1}{2} \\ \frac{x_2}{2} \\ x_1 \end{bmatrix} + \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 0 \end{bmatrix}$

affine, not linear

$$A = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \\ 1 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 0 \end{bmatrix}$$