This quiz is worth 10 points.   Name: ___Solutions___

1. (6 points) Let $A = \begin{bmatrix} 1 & 2 \\ 0 & -1 \\ 3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 1 \\ 4 & 2 \end{bmatrix}$. Evaluate each expression below or state that the expression is not defined.

(a) $2AB$

$$2\begin{bmatrix} 1 & 2 \\ 0 & -1 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ 4 & 2 \end{bmatrix} = 2\begin{bmatrix} -2+8 & 1+4 \\ -4 & -2 \\ -6+4 & 3+2 \end{bmatrix} = 2\begin{bmatrix} 6 & 5 \\ -4 & -2 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 12 & 10 \\ -8 & -4 \\ -4 & 10 \end{bmatrix}$$

(b) $2BA$ not defined

(c) A^2 not defined

(d) $AA^T = \begin{bmatrix} 1 & 2 \\ 0 & -1 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 3 \\ 2 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1+4 & -2 & 3+2 \\ -2 & 1 & -1 \\ 3+2 & -1 & 9+1 \end{bmatrix}$

$$= \begin{bmatrix} 5 & -2 & 5 \\ -2 & 1 & -1 \\ 5 & -1 & 10 \end{bmatrix}$$

2. (4 points) Let C be an $m \times n$ matrix where $C_{ij} = \begin{cases} 1 & \text{student } i \text{ is in class } j \\ 0 & \text{student } i \text{ is not in class } j \end{cases}$. Thus, the m rows
of matrix C represent m students and the n columns of C represent n classes.

(a) Let $A = CC^T$. $(m \times n)(n \times m)$
   i. What are the dimensions of A? $m \times m$

   ii. Suppose $A_{34} = 2$. Write a sentence explaining what this means in terms of students and classes.

   Student 3 and Student 4 have 2 classes in common.

(b) Let $B = C^T C$.
   i. What are the dimensions of B? $n \times n$

   ii. Suppose $B_{34} = 2$. Write a sentence explaining what this means in terms of students and classes.

   Class 2 and Class 3 have two common students.