

This quiz is worth 10 points.

Name: Solutions

1. (4 points) Let $A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 6 & 12 & 3 & -12 \\ 2 & 4 & -1 & 0 \\ -1 & -2 & 1 & -1 \end{bmatrix}$

(a) Find the null space of A . (Hint $\text{rref}(A) = \begin{bmatrix} w & x & y & z \\ 1 & 2 & 0 & -1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.)

$$w + 2x - z = 0$$

$$y - 2z = 0$$

OR

$$w = -2x + z$$

$$y = 2z$$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2x + z \\ x \\ 2z \\ z \end{bmatrix}$$

$$N(A) = \left\{ \begin{bmatrix} -2x + z \\ x \\ 2z \\ z \end{bmatrix} : x, z \in \mathbb{R} \right\}$$

(b) Find one particular vector in the null space of A . ^{nonzero}

pick $x = z = 1$.

$$v = \begin{bmatrix} -2 + 1 \\ 1 \\ 2 \cdot 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 2 \\ 1 \end{bmatrix}$$

check

$$\begin{bmatrix} 1 & 2 & 0 & -1 \\ 6 & 12 & 3 & -12 \\ 2 & 4 & -1 & 0 \\ -1 & -2 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 + 2 + 0 - 1 \\ -6 + 12 + 6 - 12 \\ -2 + 4 - 2 \\ 1 - 2 + 2 - 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \checkmark$$

2. (6 points) For each matrix below, find its determinant.

$$\begin{aligned}
 \text{(a) } B &= \begin{bmatrix} 2 & 3 & 0 \\ 1 & 1 & -2 \\ -3 & 5 & 2 \end{bmatrix} = 2 \cdot \begin{vmatrix} 1 & -2 \\ 5 & 2 \end{vmatrix} - 3 \begin{vmatrix} 1 & -2 \\ -3 & 2 \end{vmatrix} \\
 &= 2(2 - (-10)) - 3(2 - (+6)) \\
 &= 2(12) - 3(-4) = 24 + 12 \\
 &= 36
 \end{aligned}$$

$$\text{(b) } C = \begin{bmatrix} 0 & 0 & 5 & -3 \\ 4 & 1 & -1 & 4 \\ 0 & 10 & 8 & 2 \\ 0 & 0 & 0 & 2 \end{bmatrix} \quad (\text{Hint: This should be quick.})$$

$$\begin{array}{c} \curvearrowright r_1 \leftrightarrow r_2 \end{array} \begin{bmatrix} 4 & 1 & -1 & 4 \\ 0 & 0 & 5 & -3 \\ 0 & 10 & 8 & 2 \\ 0 & 0 & 0 & 2 \end{bmatrix} \begin{array}{c} r_2 \leftrightarrow r_3 \\ \longleftrightarrow \end{array} \begin{bmatrix} 4 & 1 & -1 & 4 \\ 0 & 10 & 8 & 2 \\ 0 & 0 & 5 & -3 \\ 0 & 0 & 0 & 2 \end{bmatrix} = D$$

$$\det(C) = (-1)^2 \det(D) = 4 \cdot 10 \cdot 5 \cdot 2 = 400$$