Ch10 Part 1 Matrix Multiplication

def pg 177

$A = [A_{ij}]$ is $m \times p$ matrix

$B = [B_{ij}]$ is $p \times n$ matrix

$AB = C = [C_{ij}]$ is an $m \times n$ where

entries in i^{th} row of A

$$\underset{\text{entry in } i^{th} \text{ row} + j^{th} \text{ col. of } C}{C_{ij}} = \sum_{i=1}^{p} A_{ik} B_{kj}$$

entries in j^{th} col of B

$$= A_{i1}B_{1j} + A_{i2}B_{2j} + \dots + A_{ip}B_{pj}$$

Ex

$$A = \begin{array}{c} ① \\ ② \end{array} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \qquad B = \begin{array}{cccc} & ② & & ④ \\ \end{array}$$

$$B = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 2 & 1 & -1 \\ 2 & -1 & -1 & 2 \end{bmatrix}$$

2×3 j's 3×4

$$AB = C = \begin{array}{c|cccc} & ① & ② & ③ & ④ \\ \hline ① & 7 & 2 & \overset{-1+2-3}{-2} & \overset{-2+6}{4} \\ ② & \overset{4+12}{18} & \overset{4+10-6}{8} & \overset{-4+5-6}{-5} & 7 \end{array}$$

2×4 i's

$C_{12} = (1)(1) + (2)(2) + (3)(-1) = 5 - 3 = 2$

$C_{24} = 0 - 5 + 12$

$\boxed{\text{Ex}}$ Do you see why BA is not defined?

$$BA = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 2 & 1 & -1 \\ 2 & -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$\boxed{\text{Ex}}$ Find the product by writing out the sum

$$C = \underset{A}{\begin{bmatrix} 2 & 5 \\ \pi & \sqrt{2} \end{bmatrix}} \underset{B}{\begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix}} = \begin{bmatrix} 2\cdot 1 + 5\cdot 3 & 2(-1) + 5(2) \\ \pi(1) + \sqrt{2}\cdot 3 & \pi(-1) + \sqrt{2}(2) \end{bmatrix}$$

$$= \underset{C}{\begin{bmatrix} 17 & 8 \\ \pi + 3\sqrt{2} & 2\sqrt{2} - \pi \end{bmatrix}} = \begin{bmatrix} | & | \\ A\begin{bmatrix} 1 \\ 3 \end{bmatrix} & A\begin{bmatrix} -1 \\ 2 \end{bmatrix} \\ | & | \end{bmatrix}$$

Observation: $\operatorname{col}_1(C) = \underbrace{A \cdot \operatorname{col}_1(B)}_{\text{matrix-vector product}}$

$$\operatorname{col}_2(C) = A \operatorname{col}_2(B)$$

$$\underset{m\times p \;\; p\times n}{C = AB} = \begin{bmatrix} | & | & & | \\ A\operatorname{col}_1(B) & A\cdot\operatorname{col}_2(B) & \cdots & A\operatorname{col}_n(B) \\ | & | & & | \end{bmatrix}$$

Jill's notation $\boxed{\mathrm{Col}_j(A)} \equiv j^{th}$ column of A

$$= \begin{bmatrix} A_{1j} \\ A_{2j} \\ \vdots \\ A_{mj} \end{bmatrix} = \underset{\llcorner\text{book}}{a_j}$$

$\boxed{\mathrm{row}_i(A)} = i^{th}$ row of A

$$= \begin{bmatrix} A_{i1} & A_{i2} & \cdots & A_{in} \end{bmatrix} = b_i$$

That is matrix multiplication is repeated matrix·vector mult.