Eigenvalues and Eigenvectors

Ex 1 $A = \begin{bmatrix} 1 & 2 \\ 10 & 20 \end{bmatrix}$ $\quad v = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ $\quad w = \begin{bmatrix} 1 \\ 10 \end{bmatrix}$

We observed:

$$Av = \begin{bmatrix} 1 & 2 \\ 10 & 20 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0 \cdot \begin{bmatrix} 2 \\ -1 \end{bmatrix} = 0 \cdot v$$

$$Aw = \begin{bmatrix} 1 & 2 \\ 10 & 20 \end{bmatrix}\begin{bmatrix} 1 \\ 10 \end{bmatrix} = \begin{bmatrix} 21 \\ 210 \end{bmatrix} = 21 \begin{bmatrix} 1 \\ 10 \end{bmatrix} = 21w$$

Say: Matrix A has eigenvalues $\lambda = 0$ and $\lambda = 21$ and corresponding eigenvectors $v = (2,-1)$ and $w = (1,10)$.

Ex 2 $B = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ $\quad$ eigenvalues $\lambda = 2$ and $\lambda = 3$ w/ e-vectors e_1 and e_2

In general, matrix A has eigenvalue λ with eigenvector x if $\boxed{Ax = \lambda x.}$

$$Ax = \lambda x = \begin{bmatrix} \lambda & & 0 \\ & \lambda & \\ 0 & & \ddots \; \lambda \end{bmatrix}\begin{bmatrix} | \\ x \\ | \end{bmatrix} = \lambda I_n x$$

a vector

$$(A - \lambda I_n)x = Ax - \lambda I_n x = 0$$

only interesting if $x = 0$.

$\boxed{(A-\lambda I_n)x=0 \text{ has a nonzero solution}} \equiv \boxed{\text{The matrix } A-\lambda I_n \text{ has a null space that is nontrivial}} \equiv \boxed{N(A-\lambda I_n) \neq \{0\}}$

$\equiv \boxed{\text{The matrix } A-\lambda I_n \text{ is NOT invertible}}$

$\equiv \boxed{\det(A-\lambda I_n)=0}$

<u>definition</u> The number λ is an eigenvalue of the matrix A if

$$\det(A-\lambda I_n)=0$$

Ex 1 $A=\begin{bmatrix} 1 & 2 \\ 10 & 20 \end{bmatrix}$ $A-\lambda I_2 = \begin{bmatrix} 1-\lambda & 2 \\ 10 & 20-\lambda \end{bmatrix}$

$$0 = |A-\lambda I_n| = (1-\lambda)(20-\lambda)-20 = \lambda^2-21\lambda+20-20$$

$$=\lambda^2-21\lambda = \lambda(\lambda-21).$$

So $\lambda=0$ or $\lambda=21$.

Ex 2 $B = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$, $B-\lambda I_2 = \begin{bmatrix} 2-\lambda & 0 \\ 0 & 3-\lambda \end{bmatrix}$

$0 = |B-\lambda I_2| = (2-\lambda)(3-\lambda)$. So $\lambda=2$ or $\lambda=3$.