

Eigenvalues and Eigenvectors

Ex 1 $A = \begin{bmatrix} 1 & 2 \\ 10 & 20 \end{bmatrix}$ $v = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ $w = \begin{bmatrix} 1 \\ 10 \end{bmatrix}$

We observed:

$$Av = \begin{bmatrix} 1 & 2 \\ 10 & 20 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0 \cdot \begin{bmatrix} 2 \\ -1 \end{bmatrix} = 0 \cdot v$$

$$Aw = \begin{bmatrix} 1 & 2 \\ 10 & 20 \end{bmatrix} \begin{bmatrix} 1 \\ 10 \end{bmatrix} = \begin{bmatrix} 21 \\ 210 \end{bmatrix} = 21 \begin{bmatrix} 1 \\ 10 \end{bmatrix} = 21w$$

Say: Matrix A has eigenvalues $\lambda=0$ and $\lambda=21$
and corresponding eigenvectors
 $v=(2,-1)$ and $w=(1,10)$.

Ex 2 $B = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ eigenvalues $\lambda=2$ and $\lambda=3$
w/ e -vectors e_1 and e_2

In general, matrix A has eigenvalue λ with
eigenvector x if

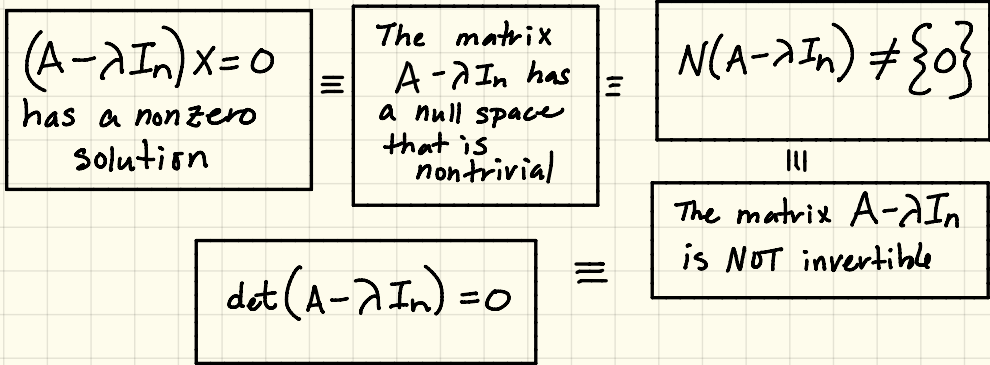
$$Ax = \lambda x.$$

$$Ax = \lambda x = \begin{bmatrix} \lambda & & 0 \\ & \lambda & \\ 0 & & \lambda \end{bmatrix} \begin{bmatrix} 1 \\ x \\ 1 \end{bmatrix} = \lambda I_n x$$

$$(A - \lambda I_n)x = Ax - \lambda I_n x = 0$$

a vector

only interesting if $x \neq 0$.



definition The number λ is an eigenvalue of the matrix A if

$$\det(A - \lambda I_n) = 0$$

Ex 1 $A = \begin{bmatrix} 1 & 2 \\ 10 & 20 \end{bmatrix}$ $A - \lambda I_2 = \begin{bmatrix} 1 - \lambda & 2 \\ 10 & 20 - \lambda \end{bmatrix}$

$$0 = |A - \lambda I_n| = (1 - \lambda)(20 - \lambda) - 20 = \lambda^2 - 21\lambda + 20 - 20$$

$$= \lambda^2 - 21\lambda = \lambda(\lambda - 21).$$

So $\lambda = 0$ or $\lambda = 21$.

Ex 2 $B = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$, $B - \lambda I_2 = \begin{bmatrix} 2 - \lambda & 0 \\ 0 & 3 - \lambda \end{bmatrix}$

$$0 = |B - \lambda I_2| = (2 - \lambda)(3 - \lambda). \text{ So } \lambda = 2 \text{ or } \lambda = 3.$$