

You

Find the eigenvalues and an

associated eigen vector for $C = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & 2 \\ -1 & 5 & 4 \end{bmatrix}$.

$$(i) \begin{vmatrix} 2-\lambda & 0 & 0 \\ 1 & 1-\lambda & 2 \\ -1 & 5 & 4-\lambda \end{vmatrix} = (2-\lambda) \begin{vmatrix} 1-\lambda & 2 \\ 5 & 4-\lambda \end{vmatrix}$$

$$= (2-\lambda)((1-\lambda)(4-\lambda) - 2 \cdot 5) = (2-\lambda)(\lambda^2 - 5\lambda - 6)$$

$$= (2-\lambda)(\lambda-6)(\lambda+1) = 0. \text{ So } \lambda = -1, 2, 6.$$

$$(ii) \lambda = -1: \begin{bmatrix} 3 & 0 & 0 \\ 1 & 2 & 2 \\ -1 & 5 & 5 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{array}{l} x_1 = 0 \\ x_2 + x_3 = 0 \end{array}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -x_3 \\ x_3 \end{bmatrix}, \quad v_1 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

check

$$\begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & 2 \\ -1 & 5 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix} \quad \checkmark$$

$$\lambda_2 = 2: \begin{bmatrix} 0 & 0 & 0 \\ 1 & -1 & 2 \\ -1 & 5 & 2 \end{bmatrix} \xrightarrow[\text{rotate rows}]{r_3 := r_3 + r_2} \begin{bmatrix} 1 & -1 & 2 \\ 0 & 4 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{r_2 := \frac{1}{4}r_2} \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{r_1 := r_1 + r_2} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} x_1 + 3x_3 &= 0 & x_1 &= -3x_3 \\ x_2 + x_3 &= 0 & x_2 &= -x_3 \end{aligned} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3x_3 \\ -x_3 \\ x_3 \end{bmatrix}$$

$$V_2 = \begin{bmatrix} -3 \\ -1 \\ 1 \end{bmatrix} \quad \text{check: } \begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & 2 \\ -1 & 5 & 4 \end{bmatrix} \begin{bmatrix} -3 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -6 \\ -3 - 1 + 2 \\ 3 - 5 + 4 \end{bmatrix} = \begin{bmatrix} -6 \\ -2 \\ 2 \end{bmatrix} \checkmark$$

$$\lambda_3 = 6: \begin{bmatrix} -4 & 0 & 0 \\ 1 & -5 & 2 \\ -1 & 5 & -2 \end{bmatrix} \xrightarrow[\text{ops}]{\text{row}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -5 & 2 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{\text{ref}}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2/5 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{So } \begin{aligned} x_1 &= 0 \\ x_2 - \frac{2}{5}x_3 &= 0 \end{aligned} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{2}{5}x_3 \\ x_3 \end{bmatrix}$$

$$V_3 = \begin{bmatrix} 0 \\ 2/5 \\ 1 \end{bmatrix} \quad \text{check: } \begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & 2 \\ -1 & 5 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ 2/5 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{2}{5} + 2 \\ 2 + 4 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{12}{5} \\ 6 \end{bmatrix} \checkmark$$