Wed Nov 13

Goal: Eigenvalues and Eigenvectors

Topics on the way: Null Spaces and Determinants

Motivating Problem: Solving a system of linear, first-order differential equations.

Ex Solve $\frac{dv}{dt} = v(t) - w(t)$ $\quad v(0) = 40$

$\frac{dw}{dt} = 2v(t) + 4w(t)$ $\quad w(0) = 10$

Claim: $v(t) = 90e^{2t} - 50e^{3t}$

$w(t) = -90e^{2t} + 100e^{3t}$

is a solution.

$\boxed{\text{You}}$ check if this is correct.

See worksheet.

Ex| Solve $\boxed{\begin{aligned}\frac{dv}{dt} &= v(t) - w(t)\\ \frac{dw}{dt} &= 2v(t) + 4w(t)\end{aligned}}$ $\boxed{\begin{aligned}v(0)&=40\\ w(0)&=10\end{aligned}}$

Claim: $\boxed{\begin{aligned}v(t) &= 90e^{2t} - 50e^{3t}\\ w(t) &= -90e^{2t} + 100e^{3t}\end{aligned}}$

How to find this? How is this Linear Algebra?

① Translate to the framework of matrices and vectors.

Let $\vec{u}(t) = \begin{bmatrix} v(t) \\ w(t) \end{bmatrix}$ ← (So $\vec{u}$ is the solution we seek)

So $\dfrac{du}{dt} = \begin{bmatrix} \frac{dv}{dt} \\ \frac{dw}{dt} \end{bmatrix}$ ← The left-hand side of the system of differential equations

Now $A = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$, Au ← The right-hand side of differential equations.

$u(0) = \begin{bmatrix} 40 \\ 10 \end{bmatrix}$ ← Initial conditions.

So this system can be written $\dfrac{du}{dt} = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix} u$, $u(0) = \begin{bmatrix} 40 \\ 10 \end{bmatrix}$

In general, these can be written $\dfrac{du}{dt} = Au$ with $u = u(0)$ at $t=0$