Ch2 Linear Functions

2.1 our defn, affine, neither

2.2 (Linear) Taylor Approx

2.3 (Linear) Regression

really interest examples you have seen in different contexts

Our functions $f: \mathbb{R}^n \to \mathbb{R}$

input a vector — output a scalar.

x - 2 vector, $x = (x_1, x_2)$

$$\underbrace{f(x) = f\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = f(x_1, x_2)}_{\text{all saying the same thing}} = 2x_1 - 3x_2 + 4$$

book writes $f(x) = 2x_1 - 3x_2 + 4$

$$f(\underbrace{3, \pi}_{\text{2-vector}}) = 2 \cdot 3 - 3 \cdot \pi + 4 = 10 - 3\pi$$ ← a number

$\boxed{\text{Worksheet here}}$

def: $f : \mathbb{R}^n \to \mathbb{R}$ is a **linear function** if

for $\boxed{\text{every}}$ pair of vectors u, v and
$\boxed{\text{every}}$ pair of scalars α, β

$$\underbrace{f(\alpha u + \beta v)}_{\text{worksheet IV}} = \underbrace{\alpha f(u) + \beta f(v)}_{\text{worksheet VII}}$$

Exs $\underbrace{1, 4 \text{ are linear}}_{\text{How do we know for SURE!!}}$, 2,3 are not

- Since $a^T(\alpha u + \beta v) = a^T(\alpha u) + a^T(\beta v)$
$= \alpha a^T u + \beta a^T v$,
we know 1 really is linear.
In fact any function written as $f(x) = a^T x$ will be linear.

- Can you rewrite #4 w/ an approiate vector a s.t. $f(x) = a^T x$?
Yes $a = \begin{bmatrix} 7 \\ -1 \end{bmatrix}$

- If $f : \mathbb{R}^n \to \mathbb{R}$ is linear, then $\exists$ some n-vector a s.t.
$$f(x) = a^T x$$

- You can figure out what "a" should be by examining $f(e_i)$.