$$\boxed{\begin{array}{llll} f:\mathbb{R}^n \to \mathbb{R} & \iff f(\alpha u+\beta v) & \iff f(x) = a^T x \\ \text{linear} & \quad = \alpha f(u) + \beta f(v) & \quad a: n\text{-vector} \\ & \text{always} & \\ & \iff f(x) = a_1x_1 + a_2x_2 + \dots + a_nx_n & a_i: \text{scalars} \end{array}}$$

So $f:\mathbb{R}^n \to \mathbb{R}$ s.t. $\boxed{f(x) = a_1x_1 + a_2x_2 + \dots + a_nx_n + \underset{\uparrow\ \text{constant}}{c}}$

is NOT linear... but ... it is AFFINE

$f:\mathbb{R}^n \to \mathbb{R}$ affine $\iff f(x) = a^T x + \underset{\uparrow\ \text{scalar}}{c} \iff$ if $\boxed{\alpha + \beta = 1}$

$\Updownarrow$

$f(\alpha u + \beta v) = \alpha f(u) + \beta f(v)$

Lots of nonlinear vector fcns that are not affine:

$$f(x) = x_1^2 + \sqrt{x_2}$$

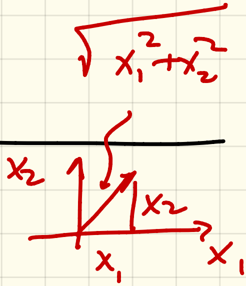
Lots of things are called "linear" that are (by our defn affine.

Eg: 1st order Taylor approx.

Ch3 Norm and Distance

[Simple applications of inner product.]

- magnitude of vector
- distance between points
- angles



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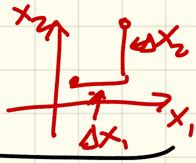
x - n vector

$$x^T x = x_1^2 + x_2^2 + \dots + x_n^2$$

$$\sqrt{x^T x} = \sqrt{x_1^2 + \dots + x_n^2} \equiv \text{magnitude of } x$$

$$\equiv \text{the norm of } x$$

$$\equiv \|x\|$$



$x = (1, 2, 3)$ $\quad \|x\| = \sqrt{1+4+9} = \sqrt{14}$

---

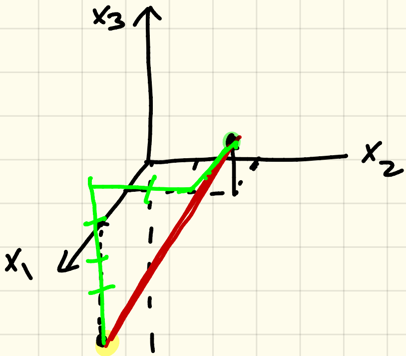
x, y n-vectors (points in n-space)

$\text{dist}(x, y)$ = distance between these points

$$= \|x - y\|$$

$$= \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + \dots + (x_n - y_n)^2}$$

$x = (1,2,1)\ ,\ y = (2,0,-3)$



$$y - x = (2-1, 0-2, -3-1)$$
$$= (1, -2, -4)$$

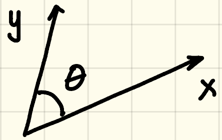
$$\|y-x\| = \sqrt{1+4+16} = \sqrt{21}$$

$$\|x-y\| = \left((x-y)^T(x-y)\right)^{\frac{1}{2}}$$

$$= \left((x-y)^T x - (x-y)^T y\right)^{\frac{1}{2}}$$

$$= \left(x^T x - y^T x - x^T y + y^T y\right)^{1/2}$$

$$= \left(\|x\|^2 - 2\, y^T x + \|y\|^2\right)^{1/2}$$



$$\cos(\theta) = \frac{x^T y}{\|x\|\|y\|}$$

$$\theta = \arccos\left(\frac{x^T y}{\|x\|\|y\|}\right)$$

What values does arccos produce (output)?