Chapter 11: Matrix Inverses (Day 1)

1. *Definition:* The matrix C is called the *left inverse* of matrix A if

   In this case, we call the matrix A *left-invertible*.

2. Suppose A is an $m \times n$ matrix. What must be the dimensions of I and C in the definition above?

3. Suppose $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 3 & 0 \end{bmatrix}$

   (a) Show that A is left-invertible by finding a left inverse C of A.

   (b) Is the left inverse, C, unique?

   (c) Is A right-invertible? Justify your conclusion.

   (d) Is A^T either left- or right-invertible?

4. Give an example of a matrix A that has at least one non-zero entry but for which no left-inverse or right-inverse exists. Give some justification that you are correct.

5. Let $S = \begin{cases} 4x + 7y = 10 \\ 2x + 6y = 20 \end{cases}$.

   (a) Write S as a matrix equation $Ax = b$.

   (b) Check that $C = \begin{bmatrix} 0.6 & -0.7 \\ -0.2 & 0.4 \end{bmatrix}$ is the left inverse of $A = \begin{bmatrix} 4 & 7 \\ 2 & 6 \end{bmatrix}$ and use C to solve the system S.

   (c) Show that C is also a right-inverse of A.

6. Let $a_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$, $a_2 = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$ and $a_3 = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$.

   (a) Write the system of equations you need to solve to show $a_1, a_2,$ and a_3 are linearly independent. (Note that you are not asked to solve this system.)

   (b) Write the system of equations from part (a) as a matrix equation $Ax = b$.

   (c) Check that the matrix $C = \begin{bmatrix} 1 & 2 & -1 \\ -2 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ is a left inverse of $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$ and use C to solve the matrix equation in part (b).

   (d) What conclusions can you draw about a matrix A if it has a left inverse C?

7. Observations