Motivation for Eigenvalues and Eigenvectors

1. An example of a system of linear, first-order differential equations.

$$\frac{dv}{dt} = v(t) - w(t) \qquad v(0) = 40$$

$$\frac{dw}{dt} = 2v(t) - 4w(t) \qquad w(0) = 10$$

2. A solution: $v(t) = 90e^{2t} - 50e^{3t}, \quad w(t) = -90e^{2t} + 100e^{3t}$.

3. Let A be an $m \times n$ matrix, then the **null space of** A is

4. Example 1: $A = \begin{bmatrix} 1 & 2 \\ 10 & 20 \end{bmatrix}$

5. Example 2: $B = \begin{bmatrix} 1 & 0 & 1 \\ 5 & 4 & 9 \\ 2 & 4 & 6 \end{bmatrix}$

6. Show/prove/give an argument for each of the statements below.

   (a) For every matrix A, $N(A) \neq \emptyset$.

   (b) If A is invertible, then $N(A)$ contains only the zero vector.

   (c) If the vector x is in $N(A)$, then for any number c, cx is in $N(A)$.

   (d) If both of the vectors x and y are in $N(A)$, then the vector $x + y$ is also in $N(A)$.

   (e) If x is in $N(A)$ and the vector z is *not* in $N(A)$, then x and z are linearly independent.

   (f) If the vector a is in $N(A)$ and c is a solution to $Ax = b$, then $c + a$ is also a solution to $Ax = b$.

   (g) If both of the vectors c_1 and c_2 are solutions to $Ax = b$, then there is a vector a in $N(A)$ such that $c_2 = c_1 + a$.

7. Main Principles