Return to thinking of the matrix-vector product as a function from $\mathbb{R}^n$ to $\mathbb{R}^m$ given an $m \times n$ matrix.

1. Example 1: Let $A = \begin{bmatrix} 1 & 2 \\ 10 & 20 \end{bmatrix}$ and $f(x) = Ax$.

   (a) State $N(A)$. (Recall that we did this on the previous sheet.)

   $N(A) = \left\{ \begin{bmatrix} -2c \\ c \end{bmatrix} : c \in \mathbb{R} \right\}$

   (b) Find the image of the vectors below under f.

   i. $v = (2, -1)$, $e_1 = (1, 0)$, $e_2 = (0, 1)$

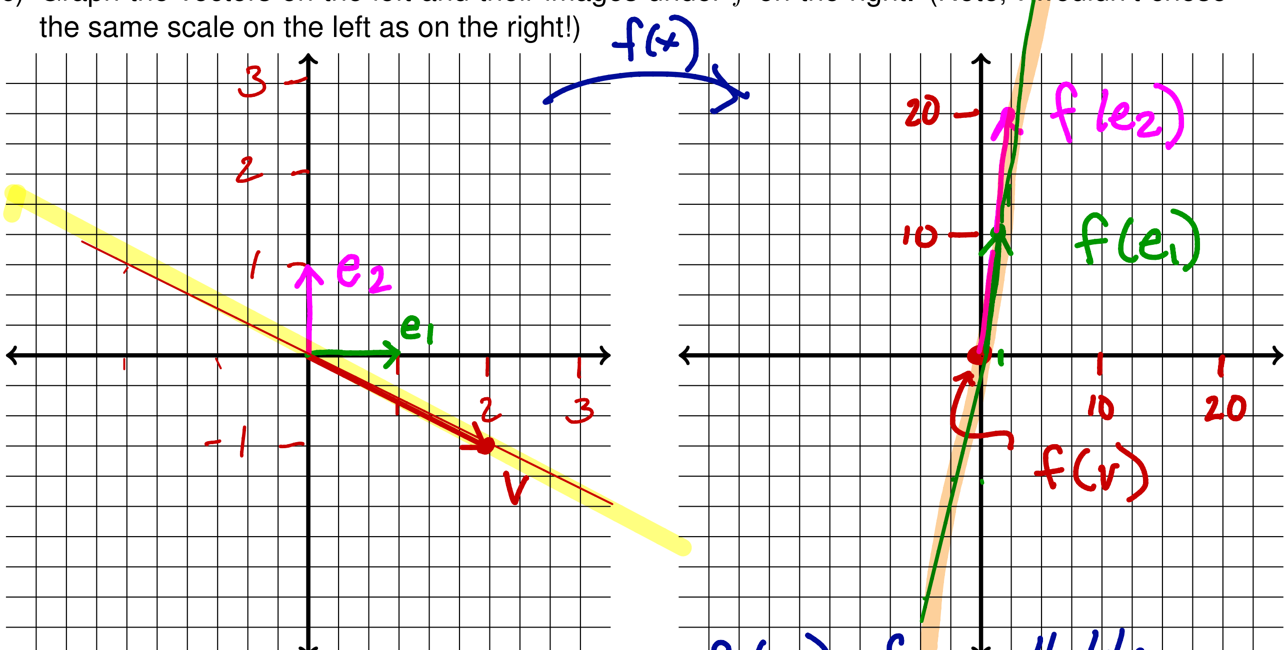
   $f(v) = \begin{bmatrix} 1 & 2 \\ 10 & 20 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

   (v in N(A)!)

   $f(1,0) = \begin{bmatrix} 1 \\ 10 \end{bmatrix}$

   $f(0,1) = \begin{bmatrix} 2 \\ 20 \end{bmatrix}$

   (c) Graph the vectors on the left and their images under f on the right. (Note, I wouldn't chose the same scale on the left as on the right!)



What can you deduce about $f(w)$ for all the other vectors w in $\mathbb{R}^2$?

- All multiples of v get sent to (0,0)
- All other vectors go to some multiple of (1,10)
- So the vector (1,10) gets sent to a multiple of itself!

$f(1,10) = \begin{bmatrix} 21 \\ 210 \end{bmatrix} = 21 \begin{bmatrix} 1 \\ 10 \end{bmatrix}$!!

2. Example 2: $B = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ and $g(x) = Bx$.

(a) State $N(B)$ $= \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$ because B is clearly invertible:

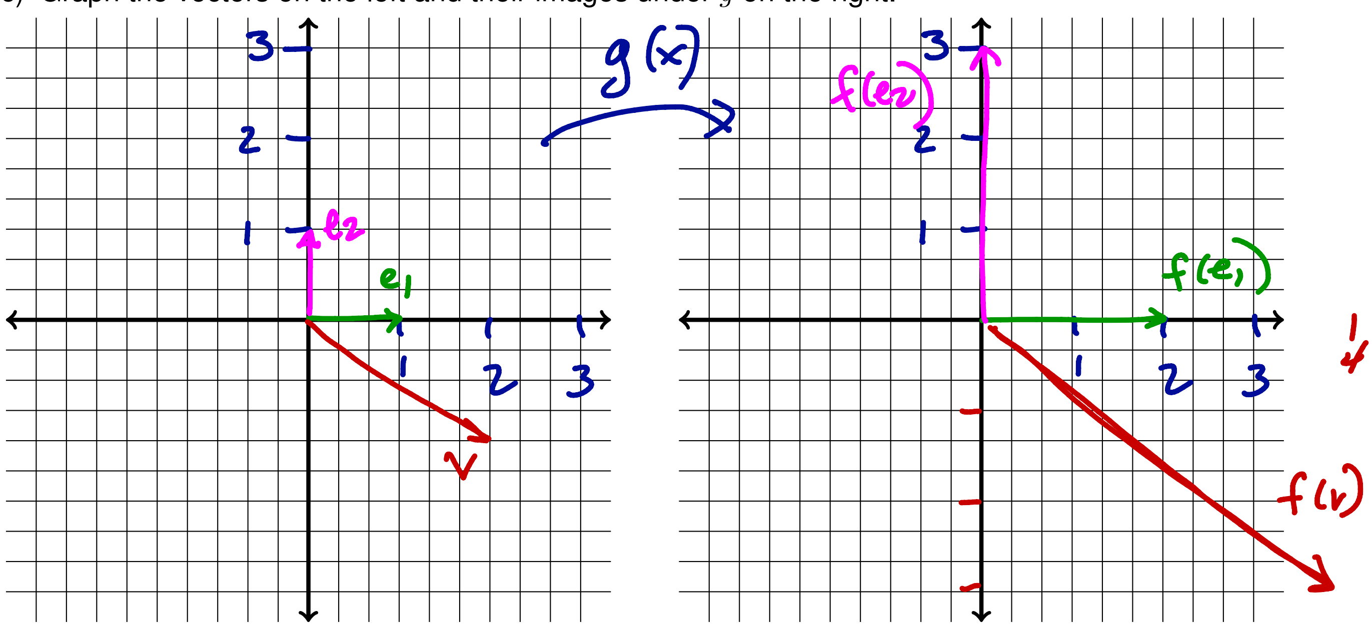
$$B^{-1} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/3 \end{bmatrix}$$

(b) Find the image of the vectors below under g.

i. $v = (2, -1)$, $e_1 = (1, 0)$, $e_2 = (0, 1)$

$f(v) = (4, -3)$, $f(e_1) = (2, 0)$, $f(e_2) = (0, 3)$

(c) Graph the vectors on the left and their images under g on the right.



What does $g(x)$ do to vectors w in $\mathbb{R}^2$?

It stretches them by a factor of 2 in the x-direction and 3 in the y-direction