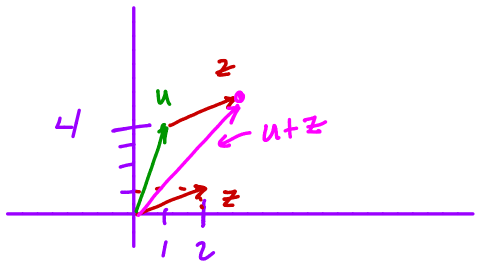
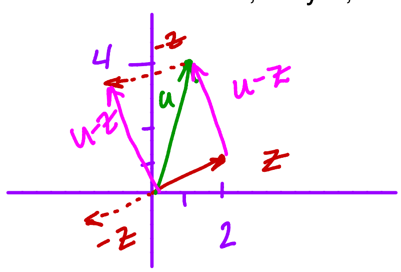
WORKSHEET: VECTOR OPERATIONS

Let $v = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$, $w = \begin{bmatrix} 4 \\ 1 \\ -1 \end{bmatrix}$, $u = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$, $z = (2, 1)$.

1. On the same set of axes, draw u, z and $u + z$.



2. On the same set of axes, dray u, z and $u - z$.



3. Make the calculations below or explain why it is not defined.

Let $v = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$, $w = \begin{bmatrix} 4 \\ 1 \\ -1 \end{bmatrix}$, $u = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$, $z = (2, 1)$.

(a) $v + u$ not defined

(b) $2v + w = \begin{bmatrix} 2 \\ -4 \\ 6 \end{bmatrix} + \begin{bmatrix} 4 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \\ 7 \end{bmatrix}$

(c) $5\,\mathbf{1}_4 - (u, u) = \begin{bmatrix} 5 \\ 5 \\ 5 \\ 5 \end{bmatrix} - \begin{bmatrix} 1 \\ 4 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 4 \\ 1 \end{bmatrix}$

(d) vw not defined

(e) $v^T w = \begin{bmatrix} 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ -1 \end{bmatrix}$

$= 1 \cdot 4 + (-2)(1) + (3)(-1) = 4 - 2 - 3$
$= -1$

(f) $w^T v = \begin{bmatrix} 4 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = 4(1) + (1)(-2) + (-1)(3)$
$= 4 - 2 - 3 = -1$

(The same as (e))!!

(g) $(w^T v)u = -1 \cdot u = (-1, -4)$

(h) $(w^T v) + u$ not defined

(i) $((w^T v), 1) + u = \begin{bmatrix} -1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$

Let $v = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$, $w = \begin{bmatrix} 4 \\ 1 \\ -1 \end{bmatrix}$, $u = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$, $z = (2, 1)$.

4. Find y_3 and $y_{2:4}$ for $y = (2v, u)$. $= (+2, -4, 6, 2, 1)$

$y_3 = 6$; $y_{2:4} = (-4, 6, 2)$

5. Suppose x is a vector of dimension 100 and $\mathbf{1} = \mathbf{1}_{100}$. Use words to describe what each calculation below will do.

(a) $\mathbf{1}^T x$ = the sum of the entries in x.

or $1^T x = \sum_{i=1}^{100} x_i$

(b) $\left(\frac{\mathbf{1}^T}{100}\right) x$ = the average of the entries in x

or $\frac{1}{100} \sum_{i=1}^{100} x_i$

(c) $\sqrt{x^T x}$ = the magnitude of vector x

$= \sum_{i=1}^{100} (x_i)^2$

(d) $(e_1 + e_2)^T x$ $= \begin{bmatrix} 1 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}^T \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{100} \end{bmatrix} = x_1 + x_2$. it's the sum of first two terms of x.

(e) Construct a vector a such that $a^T x$ gives the average of the last 10 entries in x.

$a = \frac{1}{10} \sum_{i=91}^{100} e_i$