## WORKSHEET: VECTOR FUNCTIONS

1. Define $f(x) = a^T x$ for $x = (x_1, x_2)$ and $a = (2, 4)$.

   (a) Find $f(5, -3)$

   $= \begin{bmatrix} 2 \\ 4 \end{bmatrix}^T \begin{bmatrix} 5 \\ -3 \end{bmatrix} = 10 - 12 = -2$

   (b) For $u = (5, -3)$, $v = (4, 1)$, $\alpha = 10$, and $\beta = 2$, find:

   i. $\alpha u$

   $10 \cdot (5, -3) = (50, -30)$

   ii. $\beta v$

   $= 2(4, 1) = (8, 2)$

   iii. $\alpha u + \beta v$

   $(58, -28)$

   iv. $f(\alpha u + \beta v)$

   $\boxed{\begin{bmatrix} 2 \\ 4 \end{bmatrix}^T \begin{bmatrix} 58 \\ -28 \end{bmatrix} = 116 - 112 = 4}$

   v. $\alpha f(u) = 10\left(\begin{bmatrix} 2 \\ 4 \end{bmatrix}^T \begin{bmatrix} 5 \\ -3 \end{bmatrix}\right)$

   $= 10(-2) = -20$

   vi. $\beta f(v) = 2\left(\begin{bmatrix} 2 \\ 4 \end{bmatrix}^T \begin{bmatrix} 4 \\ 1 \end{bmatrix}\right)$

   $= 2(8 + 4) = 24$

   vii. $\alpha f(u) + \beta f(v)$

   $\boxed{= -20 + 24 = 4}$

linear

2. Define $f(x) = x_1^2 + x_2^2$ for $x = (x_1, x_2)$.

   (a) Find $f(5, -3)$ $= 5^2 + (-3)^2$

   $= 25 + 9 = 34$

   (b) For $u = (5, -3)$, $v = (4, 1)$, $\alpha = 10$, and $\beta = 2$, find:

   i. $\alpha u$ $= 10(5, -3) = (50, -30)$

   ii. $\beta v$ $= 2(4, 1) = (8, 2)$

   iii. $\alpha u + \beta v$ $= (58, -28)$

   iv. $f(\alpha u + \beta v)$ $= (58)^2 + (-28)^2$

   $= 4148$

   v. $\alpha f(u) = 10(5^2 + (-3)^2)$

   $= 10(34) = 340$

   vi. $\beta f(v) = 2(4^2 + 1^2)$

   $= 2(17) = 34$

   vii. $\alpha f(u) + \beta f(v)$

   $= 340 + 34 = 374$

not linear

3. Define $f(x) = 4x_1 - x_2 + 2$ for $x = (x_1, x_2)$.

(a) Find $f(5, -3)$

$= 4(5) - (-3) + 2 = 25$

(b) For $u = (5, -3)$, $v = (4, 1)$, $\alpha = 10$, and $\beta = 2$, find:

not linear

i. $\alpha u$

$= 10(5,-3) = (50,-30)$

ii. $\beta v$

$= 2(4,1) = (8,2)$

iii. $\alpha u + \beta v$

$= (58,-28)$

iv. $f(\alpha u + \beta v)$

$= 4(58) - (-28) + 2 = 262$

v. $\alpha f(u)$

$10 \cdot 25 = 250$

vi. $\beta f(v) = 2(f(4,1))$

$= 2(4.4 - 1 + 2) = 2 \cdot 17 = 34$

vii. $\alpha f(u) + \beta f(v)$

$= 284$

(c) For $u = (5, -3)$, $v = (4, 1)$, $\alpha = 0.9$, and $\beta = 0.1$, find:

but weird?

i. $\alpha u = 0.9(5,-3)$

$= (4.5, -2.7)$

ii. $\beta v = 0.1(4,1) = (0.4, 0.1)$

iii. $\alpha u + \beta v$

$= (4.9, -2.6)$

iv. $f(\alpha u + \beta v)$

$= 4(4.9) - (-2.6) + 2 = 24.2$

v. $\alpha f(u) = 0.9(4 \cdot 5 - (-3) + 2)$

$= 0.9(25) = 22.5$

vi. $\beta f(v) = 0.1(4 \cdot 4 - 1 + 2)$

$= 0.1(17) = 1.7$

vii. $\alpha f(u) + \beta f(v)$

$= 22.5 + 1.7$

$= 24.2$

4. Define $f(x) = 7x_1 - x_2$ for $x = (x_1, x_2)$.

$$f(x) = \underbrace{\begin{bmatrix} 7 \\ -1 \end{bmatrix}^T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\text{could be written this way!}}$$

(a) Find $f(5, -3)$ $= 7 \cdot 5 - (-3) = 35 + 3 = 38$

(b) For $u = (5, -3)$, $v = (4, 1)$, $\alpha = 10$, and $\beta = 2$, find:

i. $\alpha u$

$= 10\,(5, -3) = (50, -30)$

ii. $\beta v$

$= 2(4, 1) = (8, 2)$

linear

iii. $\alpha u + \beta v$

$= (58, -28)$

iv. $f(\alpha u + \beta v)$ $= 7(58) - (-28) = 434$

v. $\alpha f(u)$ $= 10(38) = 380$

vi. $\beta f(v)$ $= 2(7 \cdot 4 - 1) = 2(28 - 1) = 54$

vii. $\alpha f(u) + \beta f(v)$ $= 380 + 54 = 434$