Why?

Suppose $\alpha+\beta=1$ and $f(x)=a^T x+c$

Let u, v be n-vectors.

Check $*$

$$f(\alpha u+\beta v)= a^T(\alpha u+\beta v) + c$$

$c = 1\cdot c = (\alpha+\beta)c$

$$= \alpha a^T u + \beta a^T v + c$$

$$= \alpha a^T u + \beta a^T v + (\alpha+\beta)c$$

$$= \alpha(a^T u + c) + \beta(a^T v + c)$$

factor α & β from like terms

$$= \alpha f(u) + \beta f(v)$$

✓

Recall Taylor Series

- Approximate $\overbrace{f(x) \text{ or } f(x,y) \text{ or } f(x,y,z)\ldots}^{\text{hard + complicated}}$ by a $\underbrace{\text{polynomial}}_{\text{simple + easy to understand}}$.

Here: LINEAR Taylor Approximation

baby ex.: $f(x) = 6e^{-x/2}$

Linear Approx of $f(x)$ at $x=0$ $\quad a=0$

formula CI-style: $\underset{f(x)}{L(x)} = f(a) + f'(a)(x-a)$

ingredients:

$f(0) = 6e^{0} = 6$

$f'(x) = 6(-\frac{1}{2})e^{-x/2} = -3e^{-x/2}$

$f'(0) = -3$

$L(x) = 6 + (-3)(x-0)$

$= 6 - 3x$

wordy interpretation?

The point: If $\boxed{b \text{ is close to } 0}$, $f(b) \approx L(b)$

Ex: Say $b = 0.1$, $L(0.1) = 6 - 0.3 = 5.7$

$f(0.1) = 5.70737\ldots$ ← calculator!

In multiple variables. Find a "linear" or affine aproximation of f for

$$f(x_1,x_2) = x_1^2 + 2x_1x_2 \quad \text{at } z=(1,3).$$

<u>Formula</u>:

$$\hat{f}(x_1,x_2) = f(z) + \frac{\partial f}{\partial x_1}(z)\cdot(x_1-z_1) + \frac{\partial f}{\partial x_2}(z)(x_2-z_2)$$

vector

$$\boxed{\hat{f}(x) = \underbrace{[\nabla f(z)]^T}_{\text{vector}}\cdot \underbrace{(x-z)}_{\text{vector}} + \underbrace{f(z)}_{\text{const.}}}$$

gradient

$$\Delta f(z) = \left(\frac{\partial f}{\partial x_1}(z), \frac{\partial f}{\partial x_2}(z), \dots, \frac{\partial f}{\partial x_n}(z)\right)$$

ingredients: $f(1,3) = 1^2 + 2\cdot1\cdot3 = 7$

$$\frac{\partial f}{\partial x_1} = 2x_1 + 2x_2, \quad \frac{\partial f}{\partial x_1}(1,3) = 2\cdot1 + 2\cdot3 = 8$$

$$\frac{\partial f}{\partial x_2} = 2x_1, \quad \frac{\partial f}{\partial x_2}(1,3) = 2\cdot1 = 2$$

$$\hat{f}(x_1,x_2) = 7 + 8(x_1-1) + 2(x_2-3)$$

<u>The point</u>: Pick vector "close" to $(1,3)$, say $(1.1,2.9)$

wordy interpretation?

$$\hat{f}(1.1,2.9) = 7+8(1.1-1)+2(2.9-3) = 7+0.8-0.2 = \underline{\underline{7.6}}$$

$$f(1.1,2.9) = (1.1)^2 + 2(1.1)(2.9) = 7.59$$