WORKSHEET: VECTOR ALGEBRA, LINEAR AND AFFINE FUNCTIONS

1. Label each of the statements below TRUE or FALSE.

   Let a, u, and v be n-vectors and let α and β be scalars.

   (a) $a^T u = u^T a$

   (b) $\alpha(u+v) = \alpha u + \alpha v$

   (c) $\alpha(a^T u) = (\alpha a)^T u$

   (d) $\alpha(a^T u) = a^T(\alpha u)$

   (e) $a^T(u+v) = a^T u + a^T v$

   (f) $\beta(a^T u) + \beta = \beta(a^T u + 1)$

2. Complete the definition of a *linear vector function*:
   The function $f : \mathbb{R}^n \to \mathbb{R}$ is linear if for every pair of vectors u and v and every pair of scalars α and β,

3. Make up two examples of functions $f : \mathbb{R}^3 \to \mathbb{R}$, one that is linear and one that is not linear.

4. Every linear function can be written

5. The definition of an *affine vector function*:
   The function $f : \mathbb{R}^n \to \mathbb{R}$ is *affine* if for every pair of vectors u and v and every pair of scalars α and β

6. Every affine function can be written

7. Linear Taylor Approximations

8. Let $f(x) = x_1 e^{-x_2} + x_3$ and $z = (2, 0, 1)$.

   (a) Find $\widehat{f}(x)$, the linear Taylor approximation of f at z.

   (b) Find $f(2.1, 0.1, 0.9)$ and $\widehat{f}(2.1, 0.1, 0.9)$.

9. Linear Regression

10. In the chart below, x_1 is house area in 1000 square feet and x_2 is the number of bedrooms. Assume the coefficient vector is $\beta = (148.73, -18.85)$ and $v = 54.40$.

House	x_1 (area)	x_2 (beds)	y (price)	$\hat{y}$ (prediction)
1	0.846	1	115.00	161.37
2	1.324	2	234.50	213.61
3	1.150	3	198.00	168.88
4	3.037	4	528.00	430.67
5	3.984	5	572.50	552.66

Write out the linear approximation $\widehat{y}$ given by β and v and confirm that the top entry in the last column is correct.

11. Interpret the coefficients in β.