1. The **norm** of x is

the norm of $v = \|v\| = \sqrt{v_1^2 + v_2^2 + \dots + v_n^2} = v^T v$

2. For n-vectors v and w and constant $\beta = -2$, find

(a) $\|v\|$

- $\|v\| = \sqrt{2^2 + (-1)^2 + 3^2} = \sqrt{4+1+9} = \sqrt{14} \approx 3.7$

(b) $\|\beta v\|$

- $\|\beta v\| = \|(-4,2,-6)\| = \sqrt{16+4+36} = \sqrt{56} = 2\sqrt{16} \approx 7.4$

(c) $\|w\|$

- $\|w\| = \sqrt{25+0+100} = \sqrt{125} = 5\sqrt{5} \approx 11.8$

(d) $\|v + w\|$

- $\|v+w\| = \sqrt{(-3)^2 + (-1)^2 + (13)^2} = \sqrt{179} \approx 13.4$

(e) $\|v - w\|$

- $\|v-w\| = \sqrt{8^2 + (-1)^2 + (-7)^2} = \sqrt{114} \approx 10.7$

3. Properties of a norm

Properties of this norm:

- $\|v\| \geq 0$
- If $\|v\| = 0$, then $v = 0$
- $\|\beta v\| = |\beta| \, \|v\| \quad (\sqrt{\beta^2} = |\beta|)$
- Δ-ineq. $\|v+w\| \leq \|v\| + \|w\|$

4. (Algebra:) For vectors x and y, and scalar α, show that $\|\alpha x + y\|^2 = \alpha^2\|x\|^2 + 2\alpha x^T y + \|y\|^2$.

$$\|\alpha x + y\|^2 = (\alpha x + y)'(\alpha x + y)$$
$$= (\alpha x)'(\alpha x) + (\alpha x)'y + y'(\alpha x) + y'y$$
$$= \alpha^2 x'x + 2\alpha x'y + y'y$$
$$= \alpha^2\|x\|^2 + 2\alpha x'y + \|y\|^2$$

5. The distance between n-vectors (points in $\mathbb{R}^n$) x and y is

$$\mathrm{dist}(x,y) = \|x-y\| \; (= \|y-x\|)$$

6. The **root-mean-square value** of the vector v is

$$= \sqrt{\frac{v_1^2 + v_2^2 + \dots + v_n^2}{n}} = \frac{\|v\|}{\sqrt{n}} = \text{(intuitively) averge of } |x_i|$$

7. **std**(v)

$$= \frac{\sqrt{(v_1 - \mathrm{avg}(v))^2 + (v_2 - \mathrm{avg}(v))^2 + \dots + (v_n - \mathrm{avg}(v))^2}}{\sqrt{n}} = \frac{\|v - (\mathrm{avg}(v))\mathbf{1}_n\|}{\sqrt{n}}$$

8. Fill in the table below

vector, v	$\|v\|$	**rms**(v)	$(\mathbf{1}^T v)/n$	**std**(v)
$(1,1,1,1)$	2	1	1	0
$(-1,1,-1,1)$	2	1	0	1
$(\sqrt{2},\sqrt{2})$	2	$\sqrt{2}$	$\sqrt{2}$	0
$(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2},1,1,1)$	2	$\frac{\sqrt{4}}{\sqrt{7}} = \frac{2}{\sqrt{7}} \approx 0.75$	$\frac{1}{3}$	$(\frac{-5}{6}, \frac{-5}{6}, \frac{1}{6}, \frac{1}{6}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3})$ $0.62994\ldots$